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The Mathematics Teacher

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The engineer in industry—seen and unseen mathematics[1]

EVERETT S. LEE, *Editor,* General Electric Review,
General Electric Company, Schenectady, New York.

Ofttimes, the people who have the original ideas and the drive to do new things are young—very young.

THIS SUBJECT was selected from an editorial of mine in the November 1953 issue of the *General Electric Review*. It was in honor of William LeRoy Emmet, a great General Electric engineer, on the fiftieth anniversary of the generators which he designed for Niagara Falls.

It was a little over 51 years ago, on October 31, 1902, that the first electric generator went on the line—that is, went to work—in Adams Station No. 2 at Niagara Falls. Ten sister machines, each rated 5000 horsepower, 250 revolutions per minute, two phase, 25 cycles, 2200 volts, followed rapidly, the last one going into service on March 16, 1904. Ever since, except when water was not available, those electric generators have continued to deliver power 24 hours a day.

After 20 years of continuous operation, one generator required repairs because of accidental mechanical damage. Since then, five have been rewound. But aside from this, all eleven machines are the same as when they left the Schenectady factory half a century ago.

The Niagara Falls Power Company gave the Adams Station No. 2 contract to General Electric " . . . for the reason that they had great confidence in W. L.R. Emmet, who had planned and would direct the work." Mr. Emmet described the occasion in these words:

> I can hardly give an idea of the triumph which I felt when this matter was settled. I was athirst for something big to do, and this was the biggest thing in the electrical world. Every detail of what I had intended to do was in my mind, as if the plant had been running for months. I had planned the construction of the generators and had them studied and calculated over and over again.

Mr. Emmet was a young man then, and you teachers of mathematics, who are privileged to work with our young people, know of the unbounded enthusiasm of youth and of what great power it is. You who are giving your lives of service to youth may well be proud as you see your pupils develop the foundations you have given them into greater, unbounded service of mankind.

It is the same today as it was in Mr. Emmet's youth. In one division at General Electric, among 767 engineers working on such products as gyroscopic gunsights, autopilots for jet fighters, bomber armament systems, naval gunfire controls, and guided missiles, the average age is 29.

Working on nuclear power for submarines and nuclear power for planes is a group of research associates, research assistants and engineers averaging less than 34 years of age.

Three young men in their twenties designed General Electric's first large-scale reactor to produce that new chemical

[1] Delivered at the Thirty-Second Annual Convention at Cincinnati, Ohio, April 22, 1954.

prodigy, silicones. For this accomplishment they received the Company's top award for outstanding achievement.

The armament system for the famous B-29 bomber was developed by a team of General Electric engineers whose average age was 26.

When we take trained young men and supply them with an experienced organization and planning, then put at their disposal resources and manufacturing know-how, the results surprise even the young men themselves.

Now in back of all of these achievements is science and mathematics. You taught these young men mathematics. You opened the doorway, you showed them what they could do with mathematics. As Mr. Emmet said, " . . . every detail of what I had intended to do was in my mind, as if the plant had been running for months. I had planned the construction of the generators and had them studied and calculated over and over again."

You have the Emmets in your classes—the boys who see ahead and who are anxious to get their mathematics, to use it, to apply it. These are the creative engineers, the producers of new things. They bring to industry the power to bring new products into being, to put improvements into present products. They are the engineers who bring us all the products we have.

And these products are astounding when we stop to think of them. Just as an example, in the short span of years from 1935 to 1950 industry brought us among other things radar and television, nylon and other synthetic fibers, the continuous-flow chemical plant, the diesel-electric and gas-turbine-electric locomotives, silicones and synthetic rubber, penicillin and antibiotics, cortisone, the jet engine, magnesium and titanium for structural use, the electronically-controlled automatic machine tool, the fork truck, the electronic computer, the germanium diode and the transistor, and nuclear power. Some of these had been discovered or were in some stage of existence before 1935; nevertheless, they were brought into widespread application in the short period of these 15 years.

Without mathematics these things would never have been.

Parallel with all of these new products were the many improvements in existing products—products which we had *before* 1935—the automobile, the railroad, the airplane, the telephone and radio, home appliances, lamps and lighting, highways and bridges and buildings and houses, the vast electric power systems, electronics, plastics and chemicals, metallurgy, instruments for measurements and control, health systems, the distribution of foods, and improvements in materials of every kind.

Without mathematics these products would not have been improved.

You can see the mathematics go into the heads of your boys and girls. Oh, yes, you have to hammer, hammer, hammer, and work, work, work to get it in there—but it does get absorbed, and as it is absorbed it begins to bear fruit. From then on it is unseen, but it is there just the same. If it were not, there would be no resulting service to mankind.

In much of your everyday life you see the mathematics beyond the classroom. You see it in figuring the bill in the grocery store, you see it in the bank, you see it when arranging a trip, you see it in making out an income tax return. You know it is there, and you can visualize it.

I want you to have the same firm conviction that mathematics is also in the host of products that you use every day and that have been brought to you by industry.

In everything that moves, there is a force. The first simple equation of mathematics surrounding that force is: $F = Ma$, which is the one-line statement of the physical law which says that the force F acting upon a mass M to give it an acceleration a is equal to the product of the mass and the acceleration. From here the engineer carries on to arrive at an end result

which gives him a picture of the motion of the particular machine on which he is working. You get it later as a product for your and everybody else's use, be it in a bicycle or a diesel-electric locomotive. The mathematics is unseen, but it is there just the same, the tool of the engineer and scientist.

You have all heard of electricity, and most of you feel you are acquainted with it. Even the little girl who wrote about Benjamin Franklin was conscious of it. She wrote: "Benjamin Franklin, born in Boston, he went to Philadelphia, there a young lady saw him, she smiled at him, he married her and discovered electricity."

Since Benjamin Franklin, the electricity of the universe has been the subject of never-ending study by our research scientists, and of ever-continuous development by our engineers to bring to mankind the services of electricity. These services are believed by some to be more revolutionary for the well-being of mankind than any other recorded material thing in human history.

In every electrical product you have, there is an electric circuit, and the fundamental equation of the electric circuit is:

$$e = ir + L\frac{di}{dt} + C\frac{de}{dt}.$$

This is the one-line expression for the law of nature that says the electromotive force impressed upon a circuit is consumed by the three voltages which are in turn consumed by resistance, by inductance, and by capacitance. The engineer starts with this equation, and you get the final product, be it a telephone or an automobile. You think of the telephone as electric and of the automobile as mechanical, but the ignition system of the automobile is electric and the equation is there also—unseen to be sure, but there just the same. And it was worked at over and over again by some engineer who started in one of your mathematics classes.

Need I go on further?

According to the theory of relativity there is an equivalence of mass and energy which Einstein wrote into the equation:

$$E = mc^2.$$

E is the energy equivalent of the mass m, and c is the velocity of light. If m is expressed in grams, and c is taken as 3×10^{10} centimeters per second, then the energy E will be in ergs; thus

$$E \text{ (ergs)} = 9\times10^{20}\ m \text{ (grams)}.$$

In order to obtain the energy in calories, this result is divided by 4.2×10^{7}. The energy change in any nuclear reaction can then be obtained from this equation by inserting for m the difference in mass between the interacting nuclei and other particles, on the one hand, and the products of the nuclear reaction, on the other hand.

Did I lose you in this discussion of Einstein's formula? If so, I did it deliberately. I just wanted you to realize that this is the sort of thing your pupils are going to traffic in after they leave you. I wanted you to realize how important, how fundamental is that groundwork of mathematics which you give them. For without it they couldn't go on to the higher things which are their destiny.

Unseen mathematics—yet this equation rocked the world and has entered us into a new age.

I began with Mr. Emmet's generators of electric power at Niagara Falls 50 years ago. I end with what we have today. The mathematics in it all is partly seen, mostly unseen. But it is there just the same.

Perhaps there is too much violence in my picture. But it would be the same if I had started with the babbling brook, for on its way to the waterfall, engineers would calculate the power available, and it would be brought to mankind through the unseen mathematics. Or I could go back years to the time when nature formed the coal from the forest, so engineers could find it and bring it to the great steam electric plants of today to produce

more and more electricity—again through unseen mathematics.

Or I could start with the wood-chopper in the forest. But as the logs rolled to the mill, the engineer would bring the machines into being to provide the lumber to build our houses and make our furniture, and give us our beautiful homes with lovely furnishings, and electric lights and electric appliances—all again from unseen mathematics.

So wherever I start, the end is the same—products for man to use. And in them is the unseen mathematics of the engineer.

As I have so often said, it is the wish, and the hope, and the prayer of the engineer that his products will be used for good and not for evil. This is for the people to decide. And thus as you bring knowledge to your boys and girls of the universe and all that is in it, may you also bring to them a recognition of how mathematics in their hands can play its part to bring forth the new and to improve the old. The results are as unseen as the mathematics, but they are beautiful.

High school teacher fellowship program

The Fund for the Advancement of Education is continuing its High School Teacher Fellowship program by offering approximately 200 fellowships to public secondary school teachers for the academic year 1955–56. The Fund hopes through these fellowships to strengthen secondary school classroom instruction and to stimulate widespread consideration of the purposes, the means, and the ends of the liberal studies in secondary education. Under this program each recipient will be enabled to devote a full academic year away from the classroom to activities designed to extend his own liberal education and to improve his teaching ability.

In contrast to previous years, each candidate's proposed program will be judged on the basis of its potential contribution to the strengthening of liberal studies in his school or school system as well as its intrinsic benefits to the candidate himself. Each superintendent supporting an application is being asked, therefore, to indicate the relevance of the candidate's proposed fellowship program to curricular and instructional problems in the liberal studies, to comment on the educational leadership qualities of the candidate, and further to indicate how the benefits of the fellowship may be expected to reflect themselves in both the individual's and the school system's program for succeeding years.

As in the past three years, individual awards will be made to able public secondary school teachers for the purpose of carrying forward a year's program of self-development. This purpose might be furthered, for example, by an imaginative program of study, reading, or other original work, possibly in conjunction with other secondary schools, colleges, or universities, government or industry.

Fellowships are available in the humanities, the social studies and the natural sciences. The responsibility for designing the year's program rests primarily upon the individual candidate. He is urged to seek advice and help from experienced persons in his own community or elsewhere. Strong preference will be given to applications where both the individual and his school system are firmly committed to a program which gives promise of having an important impact on both the enrichment of the individual teacher and the improvement of teaching in the liberal studies.

This fellowship program is open to teachers in all public secondary schools throughout the United States and its territories. Accordingly, the superintendent of any secondary school district where one or more teachers desire to make application under this announcement is requested to appoint a local committee to nominate the most appropriate candidates from that district.

In rural areas where there are a number of local secondary school systems, each serving a population of less than 2500 and each having its own superintendent, the combined area served by such systems within a county will be regarded as a "secondary school district" for the purpose of nominating candidates. In such cases, the county superintendent is requested to arrange for the appointment of the nominating committee.

Each local committee must include one school administrator, one classroom teacher, one school board member, and at least four lay citizens who are *not* employees of the school system. Recommendations of improperly constituted committees cannot be considered. The local committee may nominate the following number of candidates, depending upon the population (1950 census) of the secondary school district: for districts serving a population of 500,000 or more, six candidates; 100,000 to 500,000, four candidates; 50,000 to 100,000, two candidates; 2500 to 50,000, one candidate.

All classroom teachers in junior and senior high schools who have the necessary qualifications may enter the local competition. Eligibility

(*Continued on page 111*)

Rate of progress in learning arithmetic

ESTHER J. SWENSON, *University of Alabama, University, Alabama.*

The author develops a thesis, not too often adequately recognized by junior high school, high school, and college teachers, that the better the instruction the greater the individual differences.

LET US BEGIN with a statement on which there can be little argument: Children are different. Children are different in their present status and rates of progress in learning arithmetic just as they are different in height and weight and fingerprints. All teachers, I suppose, accept this statement as demonstrated fact.

Many teachers, however—perhaps a majority—make some assumptions about children's learning differences which are *not* sound and which ignore the facts which are known about individual differences. The first of these is the assumption made by many teachers that the wide range of pupil ability and performance in *his* particular class or classes is "very unusual." "Other classes may have wide spreads of ability and performance, but *mine* is *unusually* large." "Yes, children are different, but not usually *this* different." Few will suggest that their classes are remarkably alike; many will maintain that theirs are more different than usual.

Another assumption which seems to be common among teachers at all school levels is that the prevalent wide range of pupil performance and ability (in arithmetic or reading or any other area of the curriculum) is unnecessary and avoidable. They hold that it should not be true. Often, by implication at least, they indicate that it would not be true if someone else (perhaps the children's earlier teachers) had done a good job of teaching.

Here is Jimmie. He is a "whiz at arithmetic," his teacher says. He breezes through all regular arithmetic assignments. If he is lucky, he is given opportunities to extend his learnings beyond the common assignment for his classmates. In the same room is Sammie. He is "getting along all right," his teacher says. He doesn't "set the woods on fire" with his mathematical brilliance but he doesn't seem to have any difficulty keeping up, either. Also in the same class is Willie. His teacher describes him as "just plain hopeless" so far as arithmetic is concerned.

The teacher asks, "How can I hold the class together? How can I teach Jimmie and Sammie and the others if I have to struggle with children like Willie? His teachers in the lower grades could at least have taught him to add and subtract! What were his earlier teachers doing anyway when they were supposed to be teaching him arithmetic?" The answer to the first question is, of course, that the teacher should *not* "hold the class together" if that expression means giving all the same assignments and instruction.

It is, of course, true that the higher we go in the school system, the greater is the spread of abilities. Is it fair for teachers (or anyone else) to take the position that with adequate instruction we could *reduce* the range of individual differences among children instead of *increasing* it? The children in an eighth-grade class are more different in their arithmetic performance and abilities than they were when they were third-graders.

The answer is no. It can't be done! It should not be done! Children are different not only in their original status but also

in their rates of development. Other things being equal, they tend to get more and more different, not more and more alike. Junior high school groups tend to be more different than elementary school groups. Senior high school pupils tend to be more different than junior high school pupils.

We wish they were more alike. It would make teaching so much easier, we think. "If wishing could make it so," we think we'd like to have them more alike. On sober consideration, of course, imagine the monotony of having forty-two Sammies, all "average"! In any event, our wishing is not going to make it so. Children are different in their present knowledge of arithmetic. They are going to stay different, and, as they proceed, they are going to get to be more and more different. We may as well face that fact.

An automobile analogy

Suppose that we have two automobiles at the starting point. One of them is a refugee from a junk dealer's back lot; but it *runs*—after a fashion. We shall call it Model *Z*. When it is performing well, it can average 30 miles per hour on the open road.

The other car is a late model in excellent condition. It runs beautifully. It has power to spare. How fast this car could go is a secret, but it is certainly a 3-place number. For our illustration let us limit this car, which we shall call Model *A*, to an average speed of 60 miles per hour.

A averages 60 miles per hour. *Z* averages 30 miles per hour. They start out at the same time from a given starting point. They follow the same route of travel. At the end of one hour, how far will *A* have traveled? (60 miles.) At the end of one hour, how far will *Z* have traveled? (30 miles.) At the end of one hour, how far is *A* ahead of *Z*? (*30 miles.*) At the end of two hours, how far will *A* have traveled? (120 miles). At the end of two hours, how far will *Z* have traveled? (60 miles.) At the end of two hours, how far is *A* ahead of *Z*? (*60 miles.*) At the end of 3 hours, how far will *A* have gone? (180 miles.) At the end of 3 hours, how far will *Z* have gone? (90 miles.) How far apart are they now? (*90 miles.*) And so on. The longer they travel, the greater the distance between them. Model *A* gets farther and farther ahead. Model *Z* gets farther and farther behind.

Let us agree that this is an oversimplification of a very complex situation, but proceed anyway to draw the analogy for fast and slow learners in an arithmetic or any other mathematics class. Whether we count progress by the day or week or month or year, it will be true that the fast learner with his high-powered grasp of understandings can learn a great deal more arithmetic in the same length of time than can the slow learner. Even if the slow learner is doing his best (like the Model *Z*) he still is falling farther and farther behind the fast learner (Model *A*) who may not be using all the learning power he has. Though it might be hard to find a pair of learners who would maintain a steady 2-to-1 ratio in amount learned as suggested for the two automobiles, it would seem that there is a rough parallel between the Model *A*—Model *Z* ratio of distance traveled and the Jimmie-Willie ratio of learning achieved. Individual differences in learning do become greater and greater.

But what if the road is bad? What if the road is full of ruts and holes? Which car must slow down most to prevent breaking springs and jostling passengers? Model *A*, of course. What if the road is covered with a fine glaze of ice? Which car's speed should be reduced more? Model *A*. What if the road is hilly and full of hairpin curves? What if the road is a mire of mud? In every case, Model *A*, for all its power and speed, is held back proportionately more than Model *Z*. When traveling on bad roads, Model *A* and Model *Z* are both held to less than their proposed speeds, perhaps, but it is Model *A* which is retarded more. Therefore, on *bad* roads, the differences in progress between slow and fast cars is less than on *good* roads.

The road of instruction

Now how about the road of instruction? Does a poor instructional road have a comparable effect of reducing the differences in rates of learning arithmetic? Does a good instructional road have a comparable effect of increasing differences in pupil rates of progress? Does the analogy hold? It does.

Some examples have been given of hazards which make a highway "bad" for travel—mud, ice, curves, ruts, holes, hills. This is not a complete list. Neither can we provide here a complete list of those instructional hazards which make us classify a child's road of learning arithmetic as "bad." But we can consider a few such impediments to optimum progress.

Take the amount and variety and quality of teaching and learning aids, including books, workbooks, manipulative devices, films, and others. Certainly these affect the kind of instructional road the learner follows.

Consider the curriculum pattern. If it is characterized by flexibility and integration, the individual child's opportunities for learning arithmetic and for understanding where and how it fits with other experiences are immeasurably extended.

And, of course, we must consider the teacher. Does the teacher know arithmetic? Does the teacher understand arithmetic? Does the teacher know how to help children understand arithmetic? Does the teacher see problem-solving in arithmetic as more than getting answers by routine procedures to verbal statements of problems? Is arithmetic to the teacher a series of "this-is-how-you-do-it" procedures or does it include some of "this-is-what-it-means" and "this-is-why-it-works"?

We might consider another factor here which is not really a matter of the condition of the road, but which is certainly a driving or travel condition. When on a long trip, have you ever been routed through the downtown business district of a large city at store and office closing time? In the traffic jam does it make any difference whether your car can "do 90" on the open road? For the time being, you can stand still in a Model *A* or a Model *Z* just as well. In overcrowded classrooms we have a similar situation. The traffic problem has a good deal to do with both Jimmie's and Willie's progress. Probably, Jimmie is neglected more than Willie.

This is by no means a complete consideration of the contrasts between good and bad instructional roads which learners must travel. But perhaps we have said enough to throw some light on the question at hand, which is: How does better or poorer instruction in arithmetic influence the differences in learning progress for the learners? Does poor instruction tend to increase or to decrease the variation among the pupils in a class so far as their progress in learning arithmetic is concerned?

When is Jimmie (the Model *A* learner) apt to get the farthest ahead of Willie (the Model *Z* learner)? Just as with the automobiles, when the road is good. When the class is small enough that the teacher can give assistance to each as he needs it. When instructional materials are well chosen, when there are enough for all, when they are rich in possibilities for assisting children in discovery and understanding. When the curriculum is flexible enough to permit the children to have experiences in situations where there are real problems involving numbers to be solved; when the curriculum is such that the child is encouraged to relate his learnings in arithmetic and science and the social studies. When the teacher understands arithmetic; when the teacher understands children as individuals and as group members; when the teacher knows how to help children see sense in arithmetic and appreciate its values to them in their everyday uses of number and number relationships. These are the times, these are the situations in which Jimmie will far outstrip Willie in his progress in

arithmetic. Jimmie will get farther and farther ahead of Willie, but not because Willie is held back. Not at all! Jimmie will get farther and farther ahead because the road has been cleared of obstructions to progress. Willie will also be traveling at his best "30-mile-per-hour" rate. He will be doing closer to his best too, but his possible rate of progress is slower than Jimmie's. With each learning at a high rate *for him*, the difference between what Jimmie learns about arithmetic and what Willie learns about arithmetic will grow wider and wider.

Now let us look at the opposite situation. When do Jimmie (with his Model *A* abilities) and Willie (with his Model *Z* abilities) stay closest together? When the road is poor, just as with the automobiles. When the amount of individual attention received by each is at a minimum; when the assignment is the same for both; when Jimmie is held back to Willie's level or to one somewhere in between which fits Sammie. When everyone in the class uses the same textbook or perhaps a drill book devoid of aids to understanding; when there are few or no other instructional materials. When the curriculum pattern tends to isolate arithmetic experiences within the confines of the arithmetic period and the covers of the drill pad. When the teacher has a narrow conception of arithmetic; when the teacher knows how to get answers but not much beyond that; when the teacher thinks and acts as if all children ought to fit the same mold. These are the times, these are the situations in which Jimmie and Willie are held closest together in their growth in arithmetic learnings—held closer together not because Willie catches up with Jimmie but because they are both held to a low rate of progress.

Which situation will we choose for the children to whom we are supposed to teach arithmetic? Do we want to continue our complaints that children are not alike but *ought* to be more alike and *would* be more alike if only the teachers who preceded us had taught what they should? Or are we going to face the fact that the better our instruction is the greater the range of performance will be?

If we face this fact, we can begin to act accordingly, so let's face it. When you get a group of boys and girls who are far apart in their skills and understanding of arithmetic, don't waste your time trying to explain it away by blaming their last year's teacher. Get to work trying to help each one of those children to achieve what will be *for him* a good rate of progress. This won't make them more alike; it won't bring Willie up to Jimmie's level. But it will give Willie a chance to get the most learning possible for him and will challenge Jimmie to speed up that powerful learning motor of his.

Will recognition of the normality of varying rates of progress make teaching easier? No, probably not. At least there is no guarantee that it will. But it will make teaching more *rewarding*.

Slow travelers

What is to be done with the child who travels very slowly on the instructional road of arithmetic? Our Willie can learn arithmetic, but he learns much more slowly and covers much less ground in a given length of time than does Jimmie, the fast learner, or Sammie, the average learner. What shall we do with Willie? Shall we "flunk" him? Shall we say to him, "You will have to go back and do this over again. You will have to travel this road over again."

Is that what we would do with the slower car on the highway? Because it is far from its destination, would we advise that it be sent back to cover the last few miles over again? Is that progress? Hardly! It seems that we should, rather, seek to find some way of speeding up the progress of the slow car *from where it is*. We need to diagnose the situation. Why does Model *Z* travel so slowly?

Part of the answer has already been indicated. Model *Z* is Model *Z*. It is not

built for speed. We may accept that fact and still explore the possibilities of getting more progress out of Model *Z* than is being made at present.

Maybe this car is carrying a heavier load than it can manage without retarding its progress. A Model *T* Ford was laboring up a steep hill in South Dakota. Two of the three passengers got out and walked so old Wheezie could make the grade. With much groaning and straining and boiling, car and driver got to the top of the hill; but, with the lightened load, they *did* make it.

Maybe Willie needs to have his load lightened. Maybe he is carrying (or supposed to be carrying) common and decimal fractions, ratio, and percentage when he can only plod along when carrying a load of addition, subtraction, multiplication, and division of whole numbers. He can't carry this heavy load, so he chugs and jerks to a full stop and makes no progress at all with any part of the load. Maybe some of that load had better ride with Jimmie or Sammie for a while, so that Willie can at least be getting somewhere with the part of the load he can manage.

When Wheezie, the Model *T* Ford, had conquered the hill out there in South Dakota, and when the passengers had labored up the hill, they got in again and all went along nicely. Postponing some topics of study in arithmetic until the learner can master part of his load does not mean permanent postponement or "soft pedagogy." After Willie has been helped along through some of the rough spots so he feels and is competent with whole numbers, we can add again to his load such other topics and ideas and skills as he needs and can carry. He will make better progress in the long run with a load he *can* carry than he will if completely bogged down with a load he *cannot* carry.

Coming back to the Model *Z*, let us suppose it starts and falters, spits and chugs its way along—very slowly. We study the situation. We find that the gas line is partially clogged. A little dirt, perhaps, has got into the gas line. It is so small, but it causes *much* trouble. We get it out. Model *Z* is on its way again, not breaking any speed limits but able to make steady progress.

Could it be that Willie has an obstruction in his inner workings that is impeding his progress in arithmetic? Maybe there was some one little point that he did not understand and it keeps him from understanding many other things. If that is the case, this "little" thing is mighty big and mighty important. Let us find out what it is and get it out of his way. Maybe we went too fast for him one day. He almost had the idea and then his teacher looked at the clock and said, "Time's up. The rule is at the bottom of page 67. Learn it and then do Examples 1–20 on page 68." So close to getting the right idea. So near but so far. And Willie has "dirt in his fuel line." His progress is going to be impeded. And his trouble will get worse and worse until someone finds out what it is and removes it.

Sometimes an obstruction to Willie's learning has nothing whatsoever to do with the specific content of arithmetic. It may be an attitude he has acquired. He hasn't done well in arithmetic. He feels uncomfortable about it. He is a little ashamed of himself for not being able to do what he sees Sammie and Jimmie doing. He wishes he didn't have to do arithmetic and show his ignorance. He decides he can't do arithmetic. And when he gets to that point, he *can't*. His learning is blocked. What does he need? He needs to be helped to see that he *can* do arithmetic. Perhaps he needs to go back over some of the things he can do and which he does understand. He needs reassurance, and he needs to have some success in doing something new in arithmetic that he hasn't done before. He needs to demonstrate to himself that he *can do arithmetic*. This is not a time for punishment, for ten extra examples, for staying after school until he gets today's problems done. This is a time for encouragement, for help on the difficulty at hand, for understanding guidance in drawing

upon what Willie does know in order that he may discover ways of working from what he does know to what he does not know.

The same or different highways

Turning now instead to another question about children who have varying rates of progress in learning arithmetic, we might consider whether or not Jimmie and Sammie and Willie ought to be traveling the same highway. Should they, perhaps, be following different routes? So far we have given the general impression that they all follow the same route, the only differences being in the rate of progress and distance covered. For arithmetic in the elementary school I am going to suggest that Jimmie and Sammie and Willie should travel a common road, but that is not the whole story. Certainly they should all gain some knowledge and as much understanding as is possible for each of the meanings of numbers and the decimal number system, of the basic processes of arithmetic, of ways and means for solving problems which involve numbers and number relationships, of whole numbers and common and decimal fractions, of commonly used measures, and so on. On the other hand, there ought to be some side trips for Jimmie and some stopovers for Willie. Jimmie is making such good time along the road anyway that he can afford to take some time out for exploring interesting and informative side paths. He may wish to visit some of the historical shrines to the history of numbers. He may wish to climb the mountain of square root. These and other excursions will be to his benefit and his enjoyment.

Willie ought to have time for an occasional stopover. Motoring in the Model *Z* is more strenuous than in the Model *A*. One needs time to rest up, to consolidate one's plans, to start out afresh. Willie needs occasional stops for seeing where he is, for relating this to that, for building up a well-knit organization of that which he already knows.

Point destinations vs. whole-area destinations

Sometimes when one takes a trip, one has a particular destination at the end of the trip—Cincinnati or New York or Boston. Sometimes that is not the case at all. One may "tour New England" or "take a trip out West" for pleasure or for business. New England in general or the Pacific Coast States in general is the destination. The learner's destination in this analogy of traveling the instructional road to arithmetic is of the latter type. The child does not travel on and on and on to some future destination of skill and understanding in arithmetic. Rather, he is traveling around within the total content of arithmetic. This is an important distinction. It gives the whole trip added significance. It makes each experience important in and of itself as well as for what it leads to later. The learner's motivation is directed toward the present and the future, toward solving present problems and meeting present needs for arithmetic as well as preparing for future needs and uses and problems to be solved.

Furthermore, two travelers who start out from the same point and who seek the same final destination (a given point) may not necessarily follow the same route to get there, though they are apt to follow certain favored routes. But when the same two travelers set out to see a whole region, they are much more apt to follow different routes while seeing pretty much the same sights. So, when we say that Jimmie and Willie may well follow much of the same road in arithmetic learning, we are not limiting them to following it in exactly the same sequence. One pupil may learn column addition before he learns all the basic addition facts; the other may learn all the basic facts before he learns column addition. If certain important places in the region of arithmetic can be reached by only one route, Jimmie and Willie will both have to follow the same itinerary. The point is that wherever they go within

the field of arithmetic, children should be arriving as well as traveling. They should be achieving as well as getting ready to achieve.

Summary

Children learn arithmetic at varying rates.

They never start out at exactly the same point; but even if they did, the fast learner is bound to get farther and farther ahead of the slow learner.

Accordingly, the range of pupil performance in arithmetic grows wider and wider as children progress through the elementary school.

The widening range of abilities and performance is not necessarily due to lack of adequate instruction (although certainly inadequate instruction does exist). Rather, the better the instruction, the instructional materials, and the curriculum in general, the wider will be the range of pupil performance.

For pupils whose rate of progress in learning arithmetic is slower than we think it needs to be, we must study the causes for the delay and remove them, not just pile on a heavier load.

Those who can make rapid progress need to be challenged to make that progress and to extend their knowledge and understanding by as many fruitful explorations into arithmetical knowledge as seem desirable.

Finally, as we guide children in their travels through the land of Arithmetic, let us make sure that we have the right country in mind. Let us be wary lest we cause children to wander for another forty years in the wilderness of routine drills and mechanical manipulations of numbers and algorisms. Let us hasten the day when the arithmetical travels of *all* children will take place in a region of understanding and independent reasoning and rich experience with arithmetical content—a veritable arithmetical Land of Canaan!

Awards offered for mathematics projects

Do you have students who should be encouraged to do mathematics projects? Do they need an incentive? Would they and their school enjoy some degree of national recognition for good project work?

If the answers to these questions are affirmative, you will want to know more about the 1955 program of Science Achievement Awards. The American Society for Metals and the National Science Teachers Association (a department of NEA), combine their interest in education in making these awards available. Mathematics projects by individual or small groups of students are eligible to receive awards.

Students compete in one of eight geographical regions and in one of three grade levels: grades 7–8, 9–10, and 11–12. Entries must be mailed by May 15.

Some representative project titles from past years are: "Exploring the Binary System of Mathematics"; "Simple Celestial Determination of Longitude and Latitude"; "A Method of Locating Objects in Space"; "Building Number Systems on 2 and 6"; and "Dividing a Vacant Lot into the Maximum Number of Parking Spaces."

Student entry forms and complete rules may be obtained from the Future Scientists of America Foundation, National Science Teachers Association, 1201 Sixteenth St. N.W., Washington 6, D. C.

An NCTM committee in cooperation with FSAF has been appointed by President Marie Wilcox. Chairman is Donald Lenz of Parma, Ohio. Other members are Glenadine Gibb, Cedar Falls, Iowa; Veryl Schult, Washington, D. C.; George Hawkins, La Grange, Illinois; and Phillip Jones, Ann Arbor, Michigan.

A teaching unit on graphs

NATHAN S. WASHTON, *Queens College, New York, New York, and* ETHEL B. FRIEDMAN, *Russell Sage Junior High School, New York, New York. A unit developed by Ethel B. Friedman, under the supervision of Nathan S. Washton, which will be helpful to many mathematics teachers in the junior high school.*

Introduction

THIS IS A UNIT of work on graphic presentation, designed for a ninth-year mathematics class in the secondary schools. It is developed for students who are of average intellectual ability and who have been scheduled to the course because their ability and future outlook indicated to a guidance counselor that they could meet with a fair degree of success in a mathematics course which emphasizes algebra. The students are from middle-class homes. A majority of them plan to continue their education after graduation from secondary school.

This unit is listed in both New York City and New York State syllabi, and is designed to meet several of the essentials for functional competence in mathematics as set forth by the "Second Report of the Commission on Post War Plans of the National Council of Teachers of Mathematics."[1]

The unit is to be initiated during the second week of the second semester of ninth-year mathematics, after the class has adjusted to a new teacher and to new routines. The first week of the term and part of the second week should be devoted to a cursory review of the fundamentals of algebra studied during the previous semester, in order that the teacher become familiar with both the weaknesses and abilities of the pupils.

[1] "Second Report of the Commission on Post War Plans of the National Council of Teachers of Mathematics," THE MATHEMATICS TEACHER, XXXVIII (May 1945), pp. 197–198.

It is assumed that the students have had the following experiences prior to, or during, the first semester of ninth-year mathematics:

1. An inventory test to discover weaknesses in the four fundamental operations. A suitable test would be the *New York Computations Test on Mixed Fundamentals for Grades 7–12.*
2. Additional *meaningful* drill on a group or on an individual basis, to help overcome weaknesses disclosed by the inventory test.
3. An algebra prognosis test.
4. The following units: (a) Why mathematics is studied; (b) Mensuration as is usually studied in seventh- or eighth-year mathematics, including angle measurement and use of the protractor; (c) Language of algebra; (d) Meaning and use of signed numbers; (e) Solution of simple equations; (f) Fundamental operations with monomials and polynomials; (g) Solution of first degree equations and sets of first degree equations, including related verbal problems; (h) The formula and wide applications of formulas to life situations.

Statement of topic

The Educational Policies Commission of the National Education Association has grouped the goals of modern education in our democracy under four major objectives: the objective of self-realization; the objective of human relationship; the objective of economic efficiency; the objective

of civic responsibility.[2] From the goals thus set forth, the teacher is made aware of the emphasis on the social implications of education. The teacher should develop in each of the pupils the sound attitudes necessary for all who will soon become the consumers and producers in our society.

One prerequisite for effective citizenship in a democracy is an understanding of the basic mathematics for individual and community living. Daily living is largely dependent upon the development of the concepts and techniques of mathematics, since mathematics has become part of the very foundation of our modern society.

One reason for the study of mathematics is that in its direct application to life situations, it *could* give meaning to many areas of human endeavor. It is out of an awareness of the needs of citizens in a democracy, that it becomes essential for a teacher of mathematics to help future citizens develop both an understanding of graphic presentations and an ability to apply the arithmetic and algebraic principles involved in the proper interpretation of statistics.

A key unit in the development of effective citizens in our American democracy must be one which helps the student develop an awareness of the main guides that should be followed in collecting, graphing and interpreting data. Such a unit must help the student realize the importance of graphs and their wide use in the various phases of daily living. Consequently, this unit of work is directed toward developing in students, an appreciation of, and an ability to apply those principles of mathematics involved in the picturing of number relations.

Objectives

A. To develop an understanding of:
   1. the importance and wide use of graphs in daily living;
   2. current statistical representations found in newspapers, magazines and financial reports;
   3. the criteria for good graphs;
   4. the terminology associated with the construction and interpretation of graphs;
   5. the principles involved in graphing, in which arithmetic and algebraic processes are required;
   6. the purposes for which each type of graph is to be used;
   7. the graphing of a formula;
   8. the relationship between "linear" equations and graphs, and of the infinite number of solutions of a linear equation;
   9. the graphing of a linear equation and of a system of linear equations;
   10. the concept of slope, as the difference between two values of y divided by the difference between two corresponding values of x;
   11. graphing as one of the four ways of expressing a functional relationship. The other three ways are: verbal statements; formulas or equations; tables of values.

B. To develop an appreciation of:
   1. the widespread use of graphs in various phases of life;
   2. the graph as a pictorial representation of a functional relationship;
   3. the trends and events which graphs portray;
   4. the value of statistics, the role of graphic presentation of statistical data, and the dangers inherent in the misuse of statistics;
   5. the concept of coordinate systems;
   6. the value of neatness, accuracy and adequate labeling, as essentials to worthwhile graphing.

C. To make for social development and for such attitudes as:
   1. a desire for greater accuracy, increased neatness, higher quality workmanship;
   2. the enjoyment of collecting and graphing data;
   3. the enjoyment of working with others, during opportunities provided in and out of the classroom;
   4. the enjoyment of making contributions of experiences and knowledge to the class;
   5. the enjoyment of utilizing community resources.

D. To develop the following skills and abilities:
   1. proficiency in performing the fundamental operations of arithmetic and algebra, and in applying these operations to graphs;
   2. ability to construct and interpret graphs;
   3. ability to collect and interpret data found in daily social and economic problems, and to translate this data into the graphs which best fit both data and purpose;
   4. ability to identify good graphs found in

[2] Harry N. Rivlin, *Teaching Adolescents in Secondary Schools* (New York: Appleton-Century-Crofts, Inc., 1948), pp. 3–6.

newspapers, magazines and financial reports, based on the knowledge of the criteria for a good graph;
5. number sense required in work with rounded-off numbers;
6. ability to use coordinate systems;
7. ability to make and interpret the graph of a formula and of an equation;
8. ability to graph a linear equation by means of the slope and y-intercept;
9. ability to solve a set of equations graphically;
10. ability to solve verbal problems graphically.

Time allotment

(About six weeks, five forty-minute periods per week)

A. Launching the unit—2 lessons
B. Introduction to statistical graphs—6 lessons
C. Construction of statistical graphs—7 lessons
D. Vocabulary of statistical graphs—1 lesson
E. Graph of a formula—2 lessons
F. Coordinate systems—1½ lessons
G. Graphing a straight line—3½ lessons
H. Solving a set of linear equations graphically —2 lessons
I. Graphing curves—2 lessons
J. Review—2 lessons
K. Unit test—1 lesson

Content

A. *Introduction to statistical graphs*
   1. Comparison of various statistical graphs
      a. Pictograms
         (1) The use of pictograms in making comparisons
         (2) The representation of numbers
         (3) Reading a pictogram
         (4) Interpretation of the relationship between items[3]
         (5) Comparison of pictograms in which single figures of varying size are used, and those in which all the figures have the same size and each represents a definite quantity
      b. Bar graphs
         (1) The use of bar graphs in making comparisons
         (2) Relationship between heights (or lengths) of bars and their values
         (3) Reading and interpreting a bar graph
         (4) The single-bar or bar distribution graph, which shows the relation of parts to the whole[4]
      c. Circle graphs
         (1) The use of circle graphs in showing the relation of parts to the whole
         (2) Relationship between sectors of the circle and their values
         (3) Reading and interpreting a circle graph[5]
      d. Line graphs
         (1) The use of line graphs in indicating trends
         (2) Reading and interpreting a line graph
         (3) Prediction of future trends based on an ability to interpret a line graph[6]
         (4) Comparison of broken-line and smooth (or curved) line graphs
   2. Reasons for the common use of graphs in showing number relations in various phases of daily living
   3. Criteria for a good graph:
      (a) Analysis of current statistical representations found in newspapers, magazines and financial reports
      (b) Analysis of some dangers inherent in the misuse of statistics
B. *Construction of statistical graphs*
   1. Bar graphs
      (a) Obtaining the necessary numerical information
      (b) Rounding off numbers when necessary
      (c) Selection of most suitable form (vertical or horizontal)
      (d) Determination of scale to be used
      (e) Use of squared paper
      (f) Why scale must start at zero
      (g) Proper labeling of a bar graph
      (h) Possible topics for a bar graph: test marks; enrollment in the school since 1949; comparative sizes of stadiums; scores in athletic events[7]
   2. Circle graphs
      (a) Obtaining the necessary numerical information
      (b) Relation between each item and the whole
      (c) Conversion of fractions to percents
      (d) Finding the number of degrees in each sector of the circle
      (e) Use of protractor in measuring the central angle of each sector
      (f) Proper labeling of a circle graph
      (g) Possible topics for a circle graph: distribution of pupil population in the school; how each tax dollar is spent by the federal government; how the general organization spends its money; how a student spends his day; a typical expense budget[8]
   3. Line graphs
      (a) Obtaining the necessary numerical information

[3] Raleigh Schorling and John R. Clark, *Mathematics in Life, Basic Course* (New York: World Book Co., 1946), pp. 290–292.
[4] *Ibid.*, pp. 281–289.
[5] *Ibid.*, pp. 293–297.
[6] *Ibid.*, pp. 298–302.
[7] Isidore Dressler, *Reviewing Elementary Algebra (Ninth Year Mathematics)* (New York: Amsco School Publications, Inc., 1949), pp. 127–131.
[8] *Ibid.*, pp. 135–140.

(b) Rounding off numbers when necessary
(c) Selection of suitable vertical and horizontal scales
(d) Why the scales need not begin at zero
(e) Plotting the numerical information on squared paper
(f) Selection of most suitable form (broken or smooth lines)
(g) Possible topics for a line graph: changes in temperature readings and in barometric pressure; test marks, weight of an individual during a ten-week period; price trends in the stock market[9]

4. Selection of the graph which best fits both the data and the purpose of the graph.

C. *Vocabulary of statistical graphs*
1. New terms to be discussed as they arise, and only when discussion will help in the interpretation of statistical graphs
2. Use of "averages" for comparisons (mean, median, mode)

D. *Graph of a formula*
1. Review of the meaning of functions and the ways of representing functional relationships
2. Graph of a formula as another way of expressing a functional relationship
3. Graphing a formula
(a) Obtaining the necessary numerical information
(b) Selection of convenient scales for the axes
(c) Plotting the points on squared paper
(d) Joining the points
(e) Proper labeling of the graph of a formula[10]
4. Types of formulas to be graphed: $D = 20t$; $P = 3s$; $I = .03P$

E. *Coordinate systems*
1. Review of the signed number scale as an example of coordinates on a line
2. Extension of the system of coordinates on a line to coordinates in a plane
3. Representation of points in a plane by means of rectangular or Cartesian coordinates

F. *Graphing a straight line*
1. Relationship between a "linear" equation and the graph of a straight line
2. Graphic presentation of functions of the type $y = mx$, $y = mx + b$
3. Concept of slope
4. Slope—intercept form of a straight line
5. Introduction to parallel lines

G. *Solving a set of linear equations graphically*

H. *Graphing curves* (*parabola of the form* $y = ax^2$)

Procedure

A. *Launching the unit*
1. Film, "The Language of Graphs" (Chicago: Coronet Films),[11] to be shown in class, followed by the distribution of mimeographed study sheets pertaining to this film, as a guide to class discussion of graphs
2. Other visual aids: bulletin-board displays of graphs taken from newspapers, magazines, financial reports; table displays of booklets pertaining to graphs in various phases of daily living

B. *Introduction to statistical graphs*
1. Comparison of various statistical graphs, based on an analysis of the various graphs displayed on the bulletin board
(a) Pictograms
(b) Bar graphs
(c) Circle graphs
(d) Line graphs
2. Reasons for the use of graphs
3. Criteria for a good graph

C. *Construction of statistical graphs*
1. Bar graphs
2. Circle graphs
3. Line graphs

D. *Vocabulary of statistical graphs* (*mean, median, mode*)

E. *Graph of a formula* ($D = 20t$, $P = 3s$, $I = .03P$)

F. *Coordinate systems*
1. Review of the signed number scale as an example of coordinates on a line
2. Use of rectangular coordinates

G. *Graphing a straight line*
1. Plotting points
2. Concept of slope (to be illustrated with applications to steepness of a staircase and to steepness of a hill)
3. Slope and y-intercept form of a line
4. Slopes and parallel lines

H. *Solving a set of linear equations graphically*

I. *Graphing curves* ($A = s^2$, $y = ax^2$)

Materials and References for Teachers

Automobile Manufacturers Association, Detroit, Michigan, has compiled statistics on the production, sales, uses of automobiles, and will send this material upon request.

Bulletin-board material on graphs is available in newspapers, magazines, and financial reports.

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[9] *Ibid.*, pp. 131–135.
[10] *Ibid.*, p. 140.
[11] A. B. Diluna, R. F. Fleet, and M. K. Hathaway, *A Descriptive and Evaluative Bibliography of Mathematics Films* (available from Henry D. Syer, Boston University School of Education, 332 Bay State Road, Boston, Mass.), pp. 30–31.

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Words! Words! Words!

General numbers. Why should the word *number* receive the modifier *general*? Number itself represents a general idea; putting the adjective *general* in front of it does not make number more *general.*

Furthermore the phrase *general number* may be misleading. Usually a letter of the alphabet is used to represent such a "general number." But letters have other uses. The letter *e*, for example, represents a single definite number. Is *e* both a general number and a specific number?

Some may argue that *e* indeed is both general and specific—that context suggests the sense in which *e* is used. Few would deny the role of context in the great drama of human communication. Yet why should we needlessly risk confusion by using terms having multiple interpretations when terms with a single value are available?

Indeed, what most teachers have in mind when they use the language *general number* is the idea which the word *variable* adequately represents. So why not use *variable*?

Possibly the terminology of high school mathematics should grow up. Here is one instance where an opportunity exists for growth. —*Henry Van Engen.*

Modern trigonometry

RICHARD V. ANDREE, *University of Oklahoma, Norman, Oklahoma.*

Is your school still teaching the trigonometry of the surveyor when today's engineers, scientists, and mathematicians need to study analytical trigonometry—a study of the properties of the trigonometric functions?

WHAT DOES *your* school teach under the guise of trigonometry? Twenty-two hundred years ago Eratosthenes calculated the earth's radius, correct to within 25 miles, by measuring the angle that the sun's rays made with a vertical rod at Alexandria and at Aswan. At this time the most important aspect of trigonometry was the computation of the sides and angles of triangles. Today, engineering still demands this skill, but other phases of trigonometry have become so important that the numerical solution of triangles is only a minor section in a course in modern trigonometry. Some schools still spend one-third or more of their time on logarithms and the solution of triangles. This is not in the best interest of students who plan to enter engineering, science, or mathematics, and is certainly of little use to the student seeking a general education. Today's civilization demands a careful study of functional graphs (harmonics) and the dextrous use of identities, including the half-angle and double-angle forms. The solution of trigonometric equations and identities is vital to modern science.

The engineer of today is busy with problems that were unthought of twenty years ago. As a result of this, many shop technicians are doing work which was done by the engineer twenty years ago. The technician who is to get ahead today must be mathematically prepared to accept some of the responsibility which previously rested on the shoulders of the engineer. The engineer must be prepared to reach new horizons.

As Mr. C. J. Carlton has written,[1]

> Trigonometry is the real handmaiden of mathematics for the man in the machine shop and is equally important for the industrial electrician.

Mr. Carlton is in a position to know whereof he speaks. He is with the Trade and Technical Training Section, Ford Motor Company, Detroit, Michigan.

Last year a well-educated chemist was discussing some mathematical aspects of his chemical research with me. He said, "You know, one thing has bothered me for a long time. In this process the speed of ion interchange is proportional to $t \sin t$, but t is *time*, not an angle! How can you take the sine of a number like time? I know it works, but I've always worried about it." The answer, of course, is that the chemist took his trigonometry from a teacher who only taught it in terms of angles. One of the startling changes in modern trigonometry is that the student needs to think of the trigonometric functions of a number! I hope you will admit to your students that $\sin x$ is a meaningful concept when x is a number. (Don't insist upon saying x radians.) Modern industry, science, engineering, and electronics use the concept. Tables are constructed from the functions of a number and then later converted to angles if needed. It is very difficult to explain the graph $v = x + \sin x$ unless we admit that $\sin x$ is defined when x is a number. Some modern applications

[1] Carl J. Carlton, "Mathematics in the Skilled Trades," THE MATHEMATICS TEACHER, XLVI (January, 1953), p. 10.

demand the use of trigonometric functions of complex numbers as in $\cos(7+5i)$. Let us prepare our students for these situations which do arise.

After interviewing industrialists, scientists, engineers, technicians, and shop mechanics, my conclusion is that the trigonometry which is most needed by modern civilization is:

1. The graphs of trigonometric functions. The student must realize in graphing $y=2x+7\cos 5x$ that x need not be thought of as an angle.

2. Identities. Students should be given only one member of an identity such as

$$(1+\cos 2x)(\csc 2x-\cot 2x)$$

to simplify as much as possible, without realizing that their goal is $\sin 2x$.

3. Trigonometric equations, especially of higher degree.

4. The inverse trigonometric functions.

5. Definitions and applications involving polar coördinates, De Moivre's theorem, and complex numbers.

6. Practical problems involving equations and identities.

7. Electrical engineers and technicians need the relationships

$$\sin\theta=\frac{e^{i\theta}-e^{-i\theta}}{2i} \quad \text{and} \quad \cos\theta=\frac{e^{i\theta}+e^{-i\theta}}{2}$$

and their ramifications along with De Moivre's theorem.

If your school is still teaching the trigonometry of 250 B.C., then perhaps you should learn more about the trigonometric demands of modern civilization. Your school may use one of the several excellent texts on modern trigonometry and still be giving a 250 B.C. course by slighting these important modern phases. I hope not, because much of the difficulty students encounter in calculus, mechanics, and industrial applications is traceable directly to the lack of dexterity in the manipulation of trigonometric equations and identities.

The good ol' days

We hear so much—opinionwise—about how inadequately prepared college freshmen are today and how much better this situation used to be some years ago. One wonders whether distance in time does not cast a rosy glow over issues which were as critical then as they are now.

While browsing through the old volumes of THE MATHEMATICS TEACHER, the editor found the following in Volume 12–13 (1919–1920), page 129.

A. N. Smith of Colgate University obtained the opinions of instructors from various colleges (Universities of Chicago, Wisconsin, Dartmouth, and Cornell among others) on the problem of adequate high school preparation for college work in mathematics. Here are some of the replies.

"We have little to say in praise of the preparatory training of the freshmen. The average freshman knows very little about mathematics and what little he does know, he does not know well. His arithmetic and his algebra are usually atrocious. Whatever he can do, he does in a purely formal manner with no insight into the reasons for the process employed."

What are the underlying causes? "Too often the reason for the poor teaching is that the teacher can not impart the proper viewpoint and understanding because he does not have it himself."

What can be done about this situation? Quoting from a discussion which followed Professor Smith's presentation we find: "Personally I am very much in favor of establishing some sort of qualification test in all high-school subjects. However, just at present there are certain practical difficulties. There is a scarcity of teachers, the salaries are unattractive, etc. These questions vitally affect the small schools."

And we think our 1955 problems are new and different! Some even contend that the "twenties were better than the fifties" despite the fact that no objective data can be supplied to either prove or disprove their case.—*Henry Van Engen.*

Sebastian and the "wolf"

THEODORE C. RIDOUT, *Editorial staff of Ginn and Company, Boston, Massachusetts.*[1]

Here is a story of how a musician battled a "wolf," and how a schoolteacher fought the same fight in the classroom.

THIS IS NOT another parody on Little Red Ridinghood. It is rather a tale of how a musician battled a "wolf," and how a schoolteacher fought the same fight in the classroom.

The "wolf" we are interested in is a beast that plagued the makers of musical instruments for many centuries. Although many attacked him with vigor, and Johann Sebastian Bach laid him fairly low, the ghost of the critter still haunts our concert halls and keeps turning up in the physics laboratory. Since he has a mathematical origin, it seems fitting that we should discuss him in these pages.

The problem is something like this. You tune your violin by fifths, adjusting string tension until you hear a perfect fifth when two strings are sounded together. The fifth is a basic unit in musical tuning.

But so, also, is the octave. Take a very simple one-stringed instrument, Pythagoras' monochord. Every time we double the tension on the string we double the rate of vibration, and the tone goes up an octave. Starting at a single vibration per second, the process of doubling takes us up in geometric progression to frequencies of 2, 4, 8, 16, . . . until we reach 256, where we pause for breath and call the tone "middle C." If we double again we get "upper C," at a frequency of 512 per second, and so on, until the pitch is too high for even your dog to hear.

[1] Formerly master at Browne and Nichols School, Cambridge, Massachusetts.

According to the laws of physics, if C has a frequency of 256, the perfect fifth above it, G, will have a frequency of 256×1.5, which is 384. Here C:G = 1:1.5, or 4:6. Moreover the triad (or chord) C-E-G sounds best if the frequencies of these notes have the exact ratio 4:5:6. These combinations are pleasing to the ear because the harmonics, or overtones, as well as the fundamental tones, combine with a minimum of conflicts or undesirable beats.

Filling in, we have the following notes and their relative frequencies:

$$\begin{aligned} \text{C (frequency } k) - \text{D}(\tfrac{9}{8}k) - \text{E}(\tfrac{5}{4}k) - \text{F}(\tfrac{4}{3}k) \\ - \text{G}(\tfrac{3}{2}k) - \text{A}(\tfrac{5}{3}k) - \text{B}(\tfrac{15}{8}k) \\ - \text{C}(2k) \end{aligned}$$

This is known as a pure scale, and its tones have exact harmonic relations to the keynote. Music played on an instrument so tuned sounds rich and ethereally beautiful.

So much for theory. I will now retire to my workshop and construct a piano. Starting at a very low C (about 32 vibrations per second) I go up by perfect fifths along the musical alphabet. The notes will be C, G, D, A, E, B, F♯, C♯, G♯, E♭, B♭, F, and C. From bottom C to top C is just seven octaves. This looks like the beginning and the end of a complete and perfect keyboard, and I flatter myself I can fill in the other notes in proper ratios to make a shining row of ivories. But first I had better check my fifths.

Going up from C to G, I increased my frequency by the correct factor, 1.5. From G up to D, I again multiplied by 1.5. In all, I multiplied twelve times by this factor, so that my top C vibrates at a frequency that is $(1.5)^{12}$ times that of the bottom C, or 129.75 times the original.

But suppose I go up by octaves? Starting at the same C as before and doubling frequency until I have gone up seven octaves, I arrive at a frequency that is 2^7, or 128, times the original. I now have *two* top C's, and there is a difference of about a quarter of a semitone between them. What to do?

Various solutions have been offered. For many centuries a compromise known as the mean-tone scale was used. This can best be illustrated by arranging the twelve fifths around the dial of a clock, with C at twelve o'clock, G at one o'clock, and so on. Instead of the fifths being tuned in the correct ratio 1.5, they are tuned in the compromise ratio $\sqrt[4]{5}$, which equals 1.49535, introducing a very slight error. This error multiplied twelve times leaves a gap somewhere. Suppose I have the gap come after 11 on the clock, between F and top C on my keyboard. Instead of a normal interval of 7 semitones here, I have an interval of 7.4 semitones.

A musical composition involving this overgrown interval would be anything but harmonious, and might even howl like a wolf in the forest; hence the interval came to be known as the *quinte-de-loup*, or wolf fifth. One could play only in certain keys that did not run afoul of this beast.

Mathematicians of course came to the rescue, but their aid was not always appreciated. As far back as 1482 a Spaniard named Bartolo Rames proposed an equalized tuning in which all semitones should go up in the ratio of $\sqrt[12]{2}$, or 1.05946 times the note below. This divides the error exactly between successive intervals, slightly reducing each fifth, so that a complete set of twelve fifths can begin and end on exactly the same tones as a set of seven octaves. With the intervening notes filled in, we have a slightly imperfect but very useful tuning known as equal temperament.

This solution made little headway, in spite of various advocates, until the appearance of a mighty figure on the scene. This was none other than Johann Sebastian Bach, who proposed equal temperament for all keyboard instruments, and proceeded to tune his clavichord and harpsichord accordingly. He was thus able to play in any one of the twelve possible keys, and could modulate from one key to another without encountering any wolves.

To demonstrate the system he composed a series of twenty-four preludes and fugues, making use of all twelve keys. This was in 1722. He later repeated the process, giving us in all forty-eight pieces under the title "The Well-Tempered Clavichord." The term "well-tempered" of course refers to equal temperament. It is generally agreed that this work was chiefly responsible for the present universal use of equal temperament.

It was many generations before the mean-tone scale was abandoned in the tuning of pipe organs; and Bach was forced to compromise with the wolf at the console. For this reason his organ compositions are written only in the simpler keys.

My algebra class was naturally interested in all this. Plainly $\sqrt[12]{2}$ raised to the twelfth power is 2, so that twelve semitones will fit perfectly into an octave, forming a continuous chromatic scale. We checked the value of the radical on a slide rule. The physics department demonstrated a sound disk for us, proving to all the neighborhood that the frequencies under discussion produced a musical scale. The disk contained 8 rows of holes, corresponding to the notes of the scale, as follows: C (24 holes), D (27), E (30), F (32), G (36), A (40), B (45), C (48). When spun by an electric motor and played upon with a jet of compressed air, such a disk gives off "musical" tones approach-

ing the power of a steam calliope, and in the exact harmonic ratios of the pure scale.

My students wrote papers on such topics as the clavichord, pipe organs, orchestration, electronic instruments, acoustics, and so on. One or two whose musical background was stronger than the mathematical were somewhat shocked to find figures encroaching on the province of the Muse. These dissenters were cheered, however, to know that Robert Smith in 1759 characterized equal temperament as "extremely coarse and disagreeable," and that Helmholtz in 1852 considered that it made every note on the piano sound "false and disagreeable," and that on the organ it produced a "hellish row." Helmholtz had used just intonation, as he called it, or pure tuning, for his experimental harmonium, and like many a musical expert, became so conditioned to perfect harmonies that he found those of equal temperament very distasteful.

Thus, thanks to the Queen of Sciences, the "wolf" has now become a thing of the past, though my young daughter tells me that one or two of his cubs show up occasionally at the high-school band rehearsals.

What's new?

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John G. O'Brien, The Travelers Insurance Companies, Hartford, Connecticut. Illustrated booklet with tables; 28 pp.; free.

Developmental mathematics in the secondary schools

BENJAMIN BOLD. *Seward Park High School, New York City.*

Teachers are not only responsible for introducing students to new ideas but they must deepen old insights. Insights into the properties of the number system can be deepened in the algebra after the pupil has studied operations with polynomials.

ONE OF THE AIMS in teaching secondary school algebra is to extend and deepen the students' understanding and appreciation of the number system of mathematics. Whenever opportunities for achieving that aim present themselves, they should not be neglected.

In ninth-year algebra, when the unit on multiplication of polynomials is taught it is most desirable, for the reason mentioned above as well as for motivation, to show the relation between that algebraic process and its arithmetical equivalent. Thus, in the example

$$\begin{array}{r} 3t+2 \\ 2t+1 \\ \hline 3t+2 \\ 6t^2+4t \\ \hline 6t^2+7t+2 \end{array}$$

if $t=10$, we have the corresponding exercise in arithmetic

$$\begin{array}{r} 30+2 \\ 20+1 \\ \hline 30+2 \\ 600+40 \\ \hline 600+70+2 \end{array}$$

which is then condensed into its usual form.

It is very fruitful to rationalize the arithmetic algorithm by referring to the algebraic process. It then becomes very clear, for instance, why the second row of numbers in a multiplication example must be moved one place to the left.

In the eleventh year, the digit problem offers an excellent opportunity to enhance the pupil's understanding of our number system. When I introduce the digit problem, I place a multiplication example, such as

$$\begin{array}{r} 3\ 2 \\ 2\ 1 \\ \hline \end{array}$$

on the board. The pupil called upon to do the example usually hesitates, for he fears a trap. However, after he has completed the problem correctly, I ask why he moved the second column of numbers one place to the left. Various answers are usually forthcoming, such as, "It always gives the right answer," "Not doing that would give the wrong answer," "We were taught that way," etc. None of the answers ordinarily shows any real understanding of the principle involved. At this point the meaning of 32 as 3 tens plus 2 units is developed and the problem is done algebraically as

$$\begin{array}{r} 3t+2 \\ 2t+1 \\ \hline \end{array}$$

Methods of multiplication based upon other number systems (e.g., Roman nu-

merals) may be discussed to show the tremendous advantage of our number system. A brief history of the number system and bases other than ten may be considered at this time. With this introduction, digit problems acquire more meaning for the pupil and help to extend his understanding of the number system. I presume that, after the pupils exposed to the developmental program in the elementary grades reach the eleventh year, this particular motivation will no longer be effective, since the correct answer to the question will be generally known.

In the twelfth year, the algebraic function, $a_0x^n+a_1x^{n-1}+\cdots+a_n$ (where the coefficients a_0, a_1, $a_2 \cdots a_n$ and n are positive integers), can help the student attain a basic understanding of the properties of our number system. First, any positive integer can be represented using any base, x. Furthermore, if we use negative values of n, any decimal can also be represented.

At this point, the pupil can prove the properties of the number system. Let us consider the principle upon which "casting out nines" is based; namely that the remainder obtained by dividing the sum of the digits of any number by 9 is the same as that obtained by dividing the number itself by 9, and in particular, any number is divisible by 9 if, and only if, the sum of its digits is divisible by 9 (i.e., any number is congruent to the sum of its digits modulo 9). A simple proof based on the Remainder Theorem follows:

If $f(X)=a_0x^n+a_1x^{n-1}+\cdots+a_n$ is divided by $x-1$, the remainder is $f(1)=a_0+a_1\cdots+a_n$. When $x=10$, $f(X)$ represents any positive integer in the decimal system of notation and we can, therefore, conclude that if a number in that system is divided by 9, the remainder obtained is the same as that obtained by dividing the sum of the digits by 9. In other words, a number is divisible by 9, if, and only if, the sum of its digits is divisible by 9.

Furthermore, the above proof indicates that for any other system of notation, a similar procedure would apply. Thus, in a duodecimal system, checking can be accomplished by "casting out elevens." The number 8095, in the duodecimal system, is an integral multiple of eleven since the sum of its digits is divisible by eleven, as the reader can easily verify. However, it will be necessary, of course, to introduce two new symbols—to represent ten and eleven—in order to see that when 8095 is divided by eleven the quotient is 897 and the remainder is 0.

I have chosen one illustration from each of the major divisions of secondary algebra to indicate how that subject can illuminate arithmetical concepts and procedures. I am sure that the reader can supply many other illustrations. It is important, however, to utilize any opportunity that may present itself in algebra to strengthen the pupils' understanding and appreciation of our number system.

Words! Words! Words!

Literal number. What is a literal number? Is it a "letter used to represent a number?" If it is, then e, i, and π are literal numbers because each does represent a number. In the formula $A=bh$, however, the letters A, b, and h do not represent *a number*. Rather each represents *any number* arbitrarily selected from the class of real numbers greater than zero.

In the same way d in the expression $2d$ does not represent *a number*. Here again the letter represents any member of the class of real numbers (in ninth grade). By agreement, of course, further restrictions could be placed on the class of numbers represented by d.

When looked at this way it becomes evident that the expression *literal number* is really a synonym for *variable*. Then why not use the word *variable* from the start? Usually the term is dropped before the end of the first year's study of elementary algebra. So why introduce language having such limited usefulness? The term we really need is the word *variable*. The terminology of high school mathematics should grow up. Here is a chance for growth.

—*Henry Van Engen*

• DEVICES FOR THE MATHEMATICS CLASSROOM

Edited by Emil J. Berger, Monroe High School, St Paul, Minnesota

Curves of constant breadth[1]

Contributed by William J. Hazard, University of Colorado, Boulder, Colorado

There is exactly one curve having a constant radius, the circle; but there is an infinite number of curves of constant breadth (or constant diameter). Some of these are occasionally used as cams. Cundy and Rollett refer to them as "non-circular rollers."[2] Because of the interesting property implied by the reference to rollers and the fact that simple examples of such curves may be constructed with straightedge and compasses, they deserve at least passing attention from students of plane geometry.

Probably the simplest of this class of curves is the one illustrated in Figure 2. It may be constructed by using the vertices of an equilateral triangle as centers, and drawing arcs on each of the sides with a radius equal to the length of a side. For the particular purpose we have in mind in this article, the constant diameter AB should be chosen so that it is equal to the diameter of the circle in Figure 1. A convenient length for the diameter of both figures is 7″.

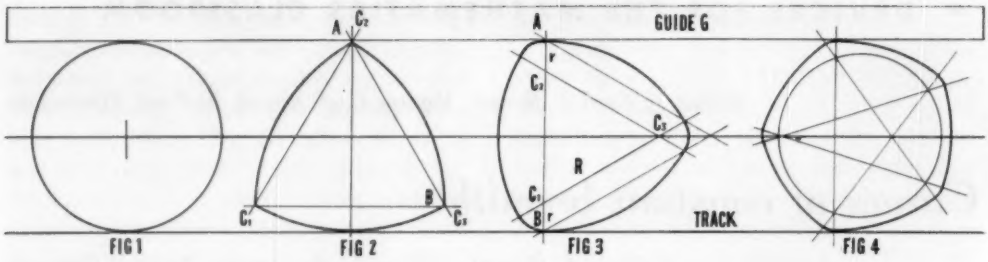
If an outline of the "puffed out triangle" illustrated in Figure 2 is reproduced on a straight-grained piece of clear pine 1″ thick and 8″ wide, and a roller cut to size (so that the constant diameter is 7″), then the resulting roller should roll smoothly. What this means is that if the guide G is pulled to the left along a line parallel to the track in such a way that the guide is in continuous contact with the roller, then the distance between the guide and the track remains constant (equal to AB). To appreciate the significance of this fact it is only necessary to observe what happens during a rotation. When the guide is moved to the left the vertex A remains at a fixed point on G until arc BC finishes rolling its length. The distance between the guide and the track during this phase of the rotation is always AB. During the second phase of the rotation C_1 is the pivot and arc AB is rolled off on G. Thus the vertices serve in succession as pivots for rolling off the opposite arcs alternately on the track and guide.

The quality of smoothness referred to above depends on the accuracy of the construction. Consequently, cutting out the rollers should be done carefully.

Before proceeding further a word about the construction of the guide seems appriate. The part labelled GUIDE G in Figures 1–4 is really only part of the guide. To perform the rolling operation efficiently two flange rails must be attached to the sides of the guide (Fig. 5). They may be made of wood and should be cut so that they extend about 1″ below the guide. The guide G may also be made of wood. It should be a trifle thicker than the rollers—just enough to allow easy passage.

[1] A curve of constant breadth is one for which the distance between any two parallel supports is always the same. The parallel supports to which we have reference are devices like the track and guide described in this article. Occasionally curves of constant breadth are also called "curves with a constant diameter."

[2] H. Martyn Cundy, and A. P. Rollett, *Mathematical Models* (London: Oxford University Press, 1952), pp. 180–183.



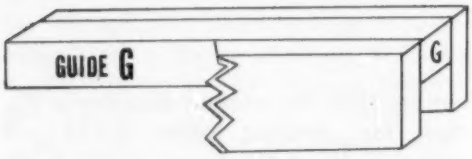


Figure 5

The TRACK is simply the level path of the roller over some plane surface such as a table top.

Figures 3 and 4 are two other curves of constant breadth; the diameters of both are equal to AB. Rollers constructed in accordance with their outlines will, when placed under the guide, support it in the manner described for Figure 2. Further, any combination of two or more of the rollers illustrated, will, when rolling together, provide smooth support for the guide.

The straightedge and compasses construction of the curve illustrated in Figure 3 is almost as simple as the one described for the curve in Figure 2. The effect of the rounded vertices is that rolling is facilitated when they are introduced. In the construction of the curve shown in Figure 3 all arcs are centered at the vertices of the triangle. To get the correct size and shape begin by drawing cord AB (equal to 7″ as before) in the position indicated and select some arbitrarily small radius r ($=AC_2$). Next lay off BC_1 equal to r. Having determined the length of C_1C_2, locate C_3 by completing equilateral triangle $C_1C_2C_3$. The three vertices are centers for all arcs on the perimeter of the curve; r and R are the radii which must be employed for drawing them. Note that $R=AB-r$.

In Figure 1, $R=r=\frac{AB}{2}$. In Figure 2, $r=0$. Thus it can be seen that for some predetermined diameter AB, r may have any length in the range O r $\frac{AB}{2}$, and that an infinite number of shapes, each depending on the chance selection of r, may be produced in the manner described.

The curve in Figure 4 may be developed by drawing a regular pentagram (star) with sides shorter than the constant diameter AB, and then choosing r and R so that $R=AB-r$ as before.

For the purposes of this article only those shapes whose constructions are based on the equilateral triangle and regular pentagon were considered. It may be of interest to the reader that it is perfectly possible to construct curves of the type described for any triangle as well as for any convex polygon with an odd number of vertices.[3]

[3] For a description of the constructions involved and a discussion of the interesting property that all curves with constant diameter AB have the length $\pi(AB)$ see Cundy and Rollett, *op. cit.*

• HISTORICALLY SPEAKING,—

Edited by Phillip S. Jones, University of Michigan, Ann Arbor, Michigan

Tangible arithmetic III: the proportional dividers

Contributed by Lucille Pinette, Colby College, Waterville, Maine

The proportional divider is a simple mathematical instrument which has been in constant use since its invention about 400 years ago. It is an interesting fact, also, that its design has remained relatively unchanged during the entire time. Figure 1 shows a proportional divider as it appeared in a book dated 1727,[1] and Figure 2 shows the divider as it appears in a modern catalog of drawing instruments.[2]

The proportional divider, or, as it is often called, the proportional compass, consists of two flat metal legs about seven inches long and one-half inch wide, grooved lengthwise and held together by a screw which permits them to open in the form of a cross. The legs terminate in a point at each end, the points at one end being about twice as long as those at the other end. On either side of the center groove are scales which vary with the use for which the instrument is intended. The two most commonly used scales are "the scale of lines" and "the scale of circles" which we will explain in some detail.

The scale of lines permits the quick and accurate reproduction of a drawing on either a reduced or enlarged scale. This scale is usually graduated for ratios from 1:1 to 1:10. If the index line on the sliding screw is set at the number 5, for example, then the distance between the longer points will be five times the distance between the shorter points. Thus lengths can be automatically either multiplied or divided by five.

One interesting special ratio which is usually designated on the scale of lines is π. By using this setting, any circle can be "rectified." That is, if the shorter points are set at the extremities of a diameter, the distance between the longer points is equal to the circumference of the circle. Thus if a draftsman wishes to construct from a sheet of paper or metal a cylinder of known diameter and if his dividers are set at π, he merely needs to open one end of them to span a distance equal to the diameter of the cylinder, and the opening at the other end represents the circumference of the cylinder's base or the length of the flat sheet which will roll up into the required cylinder.

All uses of proportional dividers depend upon the properties of similar triangles. These properties have been proved by the student of plane geometry, and they are intuitively accepted, after careful observation, even by those who are not familiar with the formal proof.

The diagram in Figure 3 illustrates some of these relations. Let AS and $A'S$ represent the longer points and SF and SF' the shorter points of a divider. The scale of

[1] Jacob Leupold, *Theatrum Arithmetica Geometricum* (Leipzig, 1727), plate XVI.

[2] *Catalogue*, Keuffel & Esser Co., 38th edition (New York), p. 112.

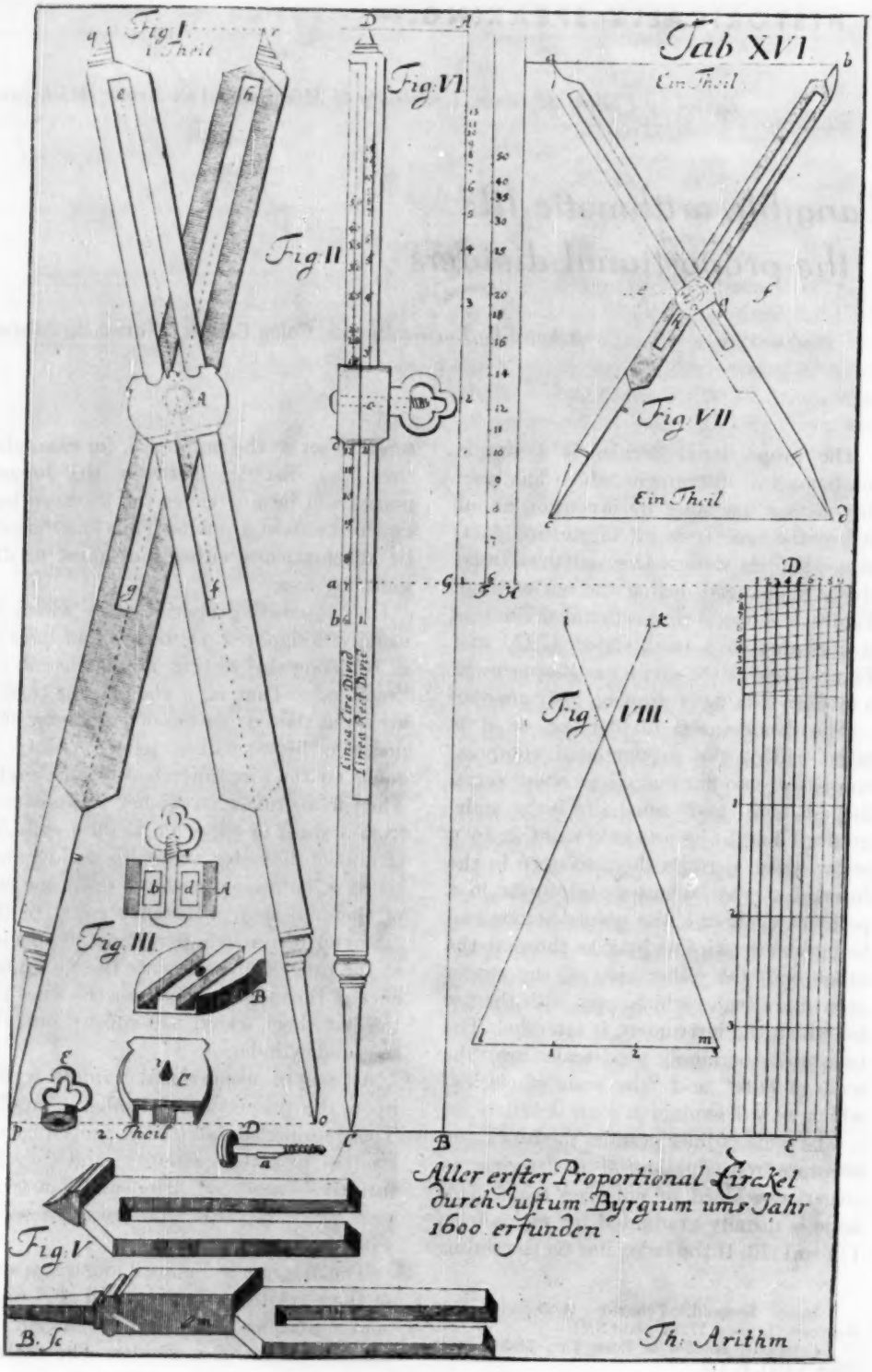

Tab XVI
Fig I
Fig II
Fig III
Fig IV
Fig V
Fig VI
Fig VII
Fig VIII
Ein Theil
2. Theil
Aller erster Proportional Zirckel
durch Justum Byrgium ums Jahr
1600 erfunden
B. sc
Th. Arithm

Figure 1

lines is graduated along AF. The triangles shown in the figure are similar. In particular, if the screw at S is set so that the ratio AS to SF is π, then the ratio AA' to $F'F$ is also π, since the triangles SAA' and $SF'F$ are similar.

The scale of lines on AF can be graduated in any set of ratios by a very simple device. Mark the length AF on a very thin sheet of paper. Then number the lines on a sheet of rather closely lined composition paper. It must be remembered that to get a ratio of, say, 4 to 1, five divisions are needed, four on one side of the required point and one on the other side.

Point A marks the end of one of the longer points of the divider. Suppose the required ratio is 4 to 1. Place the paper containing AF over the lined paper so that point A falls anywhere on line 1 and point F falls on line 6. Then the intersection of AF with line 5 determines the point dividing AF in the desired ratio, 4:1. The proof is left to the student.

One further example: suppose the desired ratio is 6 to 7. It is obvious that this requires thirteen divisions. As before, place A on line 1 and let F fall on line 14. Since A is in the direction of the longer point, then the required point falls on the intersection of line 8 with AF.

In general, it may be seen that to get a point which divides a line in the ratio p to q, it is necessary to have $p+q$ divisions. If the dividing point is designated as R, then AR is taken as either p or q divisions, whichever is larger. The reciprocal of every ratio is automatically given, simply by turning the divider.

The scale of circles is usually so graduated that the settings marked on it correspond to the ratios of the radius of a circle to the sides of inscribed regular polygons. The numbers actually engraved on the scale represent the number of sides of the polygon corresponding to the ratio determined by each setting. Such scales are usually graduated from 6 to 20.

The scale of circles is constructed by finding the central angles of as many regular polygons as are to be described on the divider. For each of these angles a ratio must be computed. To do this, construct a circle, O, of unit radius and in this circle construct a central angle equal to $360/n$, where n equals the number of sides of the polygon to be inscribed. It can be seen then in Figure 4 that if AOB is the central angle and OC its bisector, then chord AB, the length of one side of the inscribed polygon, is equal to $2 \sin \frac{1}{2}(AOB)$. Suppose this value is designated by k.

Again using the similar triangles of Figure 3, the ratio of AA' to FF', which we wish to be that of the radius of the circle to the side of the polygon, is equal to the

Figure 2

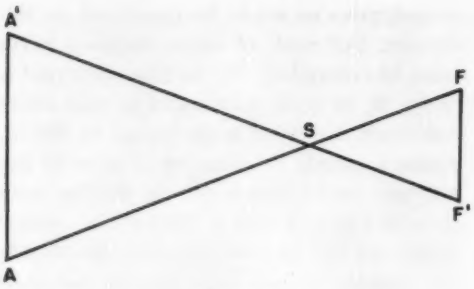


Figure 3

ratio of AS to SF. So it is necessary to know the ratio of 1 to k. Here a table of sines and a table of reciprocals would be helpful, though the calculations can be made by very careful measurement of lines in such a drawing as our Figure 4, followed by the computation of the ratios.

Suppose the figure to be inscribed is a hexagon. Then $n=6$ and the central angle is 60°. $2 \sin \frac{1}{2}\, 60° = 1$, so the required ratio is 1 to 1. Thus the midpoint of the scale AF is marked with a 6. Thus for any circle the correct ratio is maintained.

For other values of n, a similar calculation or construction may be carried out to determine the ratio. The points of the scale on the compass may then be determined by the method described for the scale of lines or by the use of a ruler. Some authors divide the scale of circles by determining the point which divides the *entire* line in the ratio of $1:k+1$. That is, if $AS:SF=1:k$, then $AS:AF=1:k+1$. In the case for $n=6$, $k+1=2 \sin \frac{1}{2}\cdot 60°+1 = 2$. The ratio being used is 1:2 in this case rather than 1:1, but the point labeled with a 6 remains the same.

A proportional divider having a fair degree of accuracy can be constructed from cardboard or other materials. The scales can be marked off by the use of a sector or by the methods mentioned above. J. Robertson, in 1747, commented "Soon after the appearance of the first proportional compasses, there were several learned and ingenious persons who contrived a great variety of scales to be put thereon. . . ."[3] Surely modern ingenuity and tools surpass those of 400 years ago; perhaps you readers and your students can devise scales for areas and volumes which, of course, are proportional to the squares and cubes of lengths, or a logarithmic scale by which the proportional dividers may be made to compute any power or root geometrically. In this latter sense the proportional divider, which may at first be thought of as a purely geometric tool, can also be viewed as a computing device and hence having a place under our general title of "Tangible Arithmetic."

One finds in tracing the history of the proportional dividers much ambiguity. This arises partly from the fact that there has been a confusion of names. One author, in referring to the proportional compass, means the sector compass, a more ambitious instrument. (See THE MATHEMATICS TEACHER, December, 1954.) Another uses the term to refer to an instrument somewhat similar to the unmarked compass used everywhere in plane geometry classrooms. In this article every reference means the 4-pointed proportional divider as described above.

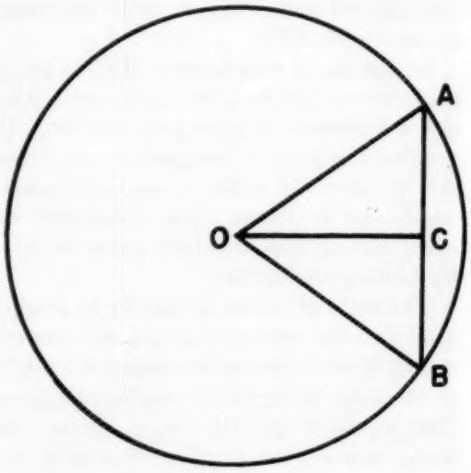


Figure 4

[3] J. Robertson, *A Treatise of such Mathematical Instruments as are usually put into a Portable Case* (London, 1747).

It has been suggested that some such instrument may have been known to Euclid. This view is probably held only by those who believe that everything geometric was known to Euclid. More reliable sources seem to point to Justus Burgius as the inventor. Again there is a variety of spellings of this name, depending upon the language in which the historian was writing. Justus Burgius is also referred to as Jost Bürgi, or J. Byrgium.

Burgius, a Swiss mathematician and instrument maker, who also, independently of Napier, invented and published a table of logarithms in 1620, has been credited by E. Voellmy, a recent biographer,[4] with the invention in 1592 of the "Proportionalzirckels," another name for the divider.

It is interesting that Leonardo da Vinci (1452–1519) has sketches of proportional dividers in several places in his sketchbooks.[5] So varied were the interests and abilities of this great master, that it is difficult to know whether he was making a sketch of an instrument which he had never seen, or was merely "doodling" as he considered some problem in mechanics for which such a device could be used. At any rate *he* makes no claim to being its inventor, although historians have often listed it as among his achievements.

J. Robertson, in his very interesting little book, published in London in 1747, *A Treatise of such Mathematical Instruments as are usually put into a Portable Case,* summarizes the story of the invention of the dividers by quoting several authorities. He says on page XV: "Christian Wolfius, in his Math lexicon, 8 vo. printed at Leipsic, an. 1716, under the word *cincinus proportionum,* relates that Levinus Hulsius, in his treatise on the proportional compasses, printed at Frankfort the 10th of May, 1603, says, that he first saw the said instrument at Ratisbon, on the day of the imperial dyet: That he had sold them far and near before 1603; and that it had been inaccurately copied in several places: Wolfius says farther, that Justus Burgius was certainly the inventor, but used to let his inventions lye unpublished.

"He then relates the contest between Galilaeus and Capra, and ends with shewing the difference between the instruments of Burgius and Galilaeus."

The reference to the instrument of Galilaeus means the previously mentioned sector compass.

Robertson also quotes Daniel Speckle who in the year 1589 published in folio his *Military Architecture* at Strasburg where he was an architect: " . . . He takes notice of compasses then in use of a curious invention, whose center could be moved forwards or backwards, so that by the figures and divisions marked thereon, a right line could be readily and correctly divided into any number of equal parts, not exceeding 20. This instrument has been since called the *proportional compasses.*"[6]

[4] E. Voellmy, "Jost Bürgi und die Logarithmen," *Elemente der Mathematik,* Beiheft Nr. 5, Oktober 1948.

[5] Leonardo da Vinci, *Problèmes de Géométrie et D'Hydraulique, Les Solides d'Égal Volume* (Paris, 1901), Édouard Rouveyre, Éditeur, Feuillet no. 4. Also, G. Boffito, *Gli Strumenti Della Scienza e la Scienza Degli Strumenti* (Firenze, 1929), p. 71, also Tavole 29.

[6] J. Robertson, *op. cit.*

"A man who doesn't take his place in society's activities is not only useless but also dangerous."—*Plato*

• MATHEMATICAL MISCELLANEA

Edited by Paul C. Clifford, State Teachers College, Montclair, New Jersey, and Adrian Struyk, Clifton High School, Clifton, New Jersey

Streamlining the proof of the binomial theorem

by Robert Mirsky, Cornell Aeronautical Laboratory, Inc., Buffalo, New York

The proof of the binomial theorem for any positive integral index is usually given in a first-year course of college mathematics. The proof is by induction and is strikingly the same in hundreds of textbooks. Because the traditional proof is wordy and unduly complicated, it is hoped that the following proof will find its way into many classrooms.

After the student has been introduced to combinatorial notation, it can be asserted that

$$(a+x)^n = C_0^n a^n x^0 + C_1^n a^{n-1} x^1 + C_2^n a^{n-2} x^2 + \cdots + C_{n-1}^n a^1 x^{n-1} + C_n^n a^0 x^n. \tag{1}$$

Verifying (1) for $n=1$, it is then necessary to show that for $n+1$ we will get

$$(a+x)^{n+1} = C_0^{n+1} a^{n+1} x^0 + C_1^{n+1} a^n x^1 + C_2^{n+1} a^{n-1} x^2 + \cdots + C_n^{n+1} a^1 x^n + C_{n+1}^{n+1} a^0 x^{n+1}. \tag{2}$$

With the assumption that (1) is true and since $(a+x)^{n+1} = (a+x)^n(a+x)$ we obtain an expression for $(a+x)^{n+1}$ by multiplying the right-hand member of (1) by $a+x$. The difficult part of the proof is to show that this product is actually equal to the right-hand member of (2).

We suggest the following demonstration, making use of a well-known lemma.

Lemma: For any non-negative integers n and r

$$C_r^n + C_{r+1}^n = C_{r+1}^{n+1}.$$

Verification is easy and is an interesting exercise for the student.

Now performing the multiplication systematically, we have

$$\begin{array}{l} (a+x)^n = C_0^n a^n x^0 + C_1^n a^{n-1} x^1 + C_2^n a^{n-2} x^2 + \cdots + C_{n-1}^n a^1 x^{n-1} + C_n^n a^0 x^n \\ \quad a+x = \\ \hline C^n a^{n+1} x^0 + C_1^n a^n x^1 + C_2^n a^{n-1} x^2 + \cdots + C_{n-1}^n a^2 x^{n-1} + C_n^n a^1 x^n \\ \qquad\qquad C_0^n a^n x^1 + C_1^n a^{n-1} x^2 + \cdots + C_{n-2}^n a^2 x^{n-1} + C_{n-1}^n a^1 x^n + C_n^n a^0 x^{n+1} \\ \hline (a+x)^{n+1} = C_0^n a^{n+1} x^0 + (C_0^n + C_1^n) a^n x^1 + (C_1^n + C_2^n) a^{n-1} x^2 + \cdots \\ \qquad\qquad + (C_{n-2}^n + C_{n-1}^n) a^2 x^{n-1} + (C_{n-1}^n + C_n^n) a^1 x^n + C_n^n a^0 x^{n+1} \end{array} \tag{3}$$

With the aid of the lemma, it is immediately apparent that (3) is equal to (2). One may perhaps note in passing that

$$C_0^n = C_0^{n+1} = C_n^n = C_{n+}^{n+1} = 1.$$

The above method of proof was used by the author, with excellent results, in a course given at Columbia University.

Editor's Note: Some books—all too few, however—do use the approach suggested by the author of this article. See for example, M. Richardson, *College Algebra* (New York: Prentice-Hall, Inc., 1947), p. 327.

Congruent triangles (fifth case) and the theorem of Lehmus[1]

by Victor Thébault, Tennie, Sarthe, France

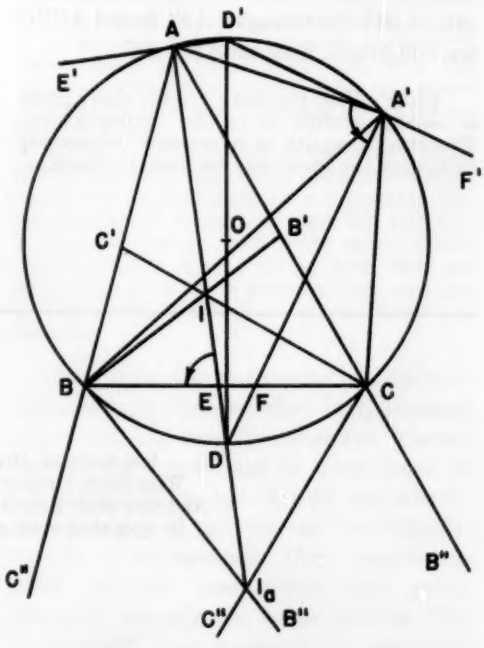
Here is a very simple direct proof of a theorem which may be considered to constitute the fifth case of the congruence of triangles,[2] because of its applications. It leads to a proof of the celebrated proposition of Lehmus—*A triangle which has two equal interior angle-bisectors is isosceles*—without using material from the second book of geometry.

Theorem. Two triangles having a side of one equal to a side of the other, and equal angles opposite these equal sides, and equal bisectors of the equal angles, are congruent.

Lemma. In a circle (O) triangles BCA and BCA' are two arbitrarily inscribed triangles, and D is the midpoint of one arc BC. If the lines DA and DA' meet BC at E and F respectively, then the quadrilateral $AEFA'$ is cyclic. (See accompanying figure.)

Proof: The angles BEA and $DA'A$ are measured by $\frac{1}{2}(\widehat{AB}+\widehat{DC})$ and $\frac{1}{2}(\widehat{AB}-\widehat{BD})$ respectively, and hence are equal, since D is the midpoint of arc BC by hypothesis. Hence quadrilateral $AEFA'$ is cyclic. The quadrilateral $AE'F'A'$, which corresponds to the point D' diametrically opposite D on circle (O), and to the points of intersection of line BC with lines $D'A$ and $D'A'$, is also cyclic.

Consider now, in the figure, the triangles BCA and BCA'. They have the side BC in common, vertices A and A' on the same side of BC, and equal angles at A and A'. If the bisectors AE and $A'F$ of these equal



[1] Translated from the French by A. Struyk.

[2] The *fourth case* of the congruence of triangles may be described as follows: *Two sides and a right or obtuse angle not included by them, in one triangle, equal respectively to two sides and the corresponding angle, in another triangle.* This has enabled us to obtain another proof of the theorem of Lehmus. See *L'Education Mathématique* (Paris), 34e année, p. 65.

angles are equal, then the chords AA' and EF of circle $AEFA'$ are parallel, and triangles DAA' and DEF are isosceles. Then triangles BCA and BCA', symmetrically situated with respect to the common perpendicular bisector of BC, EF, AA', are congruent. The same conclusion is obtained if the exterior bisectors AE' and AF' of triangles BCA and BCA' are equal, for the chords AA' and $E'F'$ of circle $AE'F'A'$ are then parallel. Thus the theorem is proven.

Corollaries. 1. *A triangle which has two equal interior angle-bisectors is isosceles.* For, if the interior bisectors BB', CC' of triangle ABC, which intersect at I on the interior bisector of angle BAC, are supposed equal, then triangles ABB' and $AC'C$ are congruent (by the fifth case), making $AB = AC$.

2. *A triangle which has two equal exterior angle-bisectors, the feet of which are not situated on opposite sides of the third side, is isosceles.* Indeed, if the exterior bisectors BB'' and CC'' of triangle ABC, which intersect at I_a on the interior bisector of angle BAC, are supposed equal, then, with B'' and C'' lying on the same side of BC, the triangles $AB''B$ and $AC''C$ are congruent, making $AB = AC$.

Editor's Note: Corollary 1 in the above paper is usually referred to as the Lehmus-Steiner Theorem. A wealth of information concerning its history and proof may be found in MacKay, David L., "The Lehmus-Steiner Theorem," *School Science and Mathematics*, Vol. 39 (June 1939), pp. 561–572. Besides historical material there are 28 proofs, direct and indirect, classified according to prerequisite theorems; and 25 bibliographical items are an addition "to the thirty odd references given by J. S. Mackay in the *Proceedings of the Edinburgh Mathematical Society*, Vol. 30, 1901–1902, pp. 18–22."

For other interesting aspects see the following:

Bleick, W. E. "Angle Bisectors of an Isosceles Triangle," *American Mathematical Monthly*, Vol. 55 (Oct. 1948), p. 495.

Sevier, F. A. C. "A New Proof of an Old Theorem," The Mathematics Teacher, Vol. XLV (Feb. 1952), pp. 121–122.

Thébault, V. "The Theorem of Lehmus," *Scripta Mathematica*, Vol. 15 (March 1949), pp. 87–88.

American Mathematical Monthly, Problem Department—Vol. 51 (Dec. 1944), pp. 590–591, E 613. Vol. 57 (Aug.–Sept. 1950), pp. 486–487, E 863.

School Science and Mathematics, Problem Department—Vol. 33 (Oct. 1933), pp. 781–783, #1283.

Corollary 2 above is related to the so-called Pseudo-Isosceles Triangle, a scalene triangle of which two exterior angle-bisectors are equal. Many properties of such a triangle are derived in:

MacKay, David L. "The Pseudo-Isosceles Triangle," *School Science and Mathematics*, Vol. 40 (May 1940), pp. 464–468.

Additional references are

Corliss, J. J. "If Two External Bisectors Are Equal Is the Triangle Isosceles?" *School Science and Mathematics*, Vol. 39 (Nov. 1939), pp. 732–735.

School Science and Mathematics, Vol. 31 (April 1931), pp. 465–466, prob. 1148.

The Pentagon, Vol. 13 (Fall 1953), pp. 35–36, prob. 34.

—*A. Struyk.*

" . . . But masters, strong and wise
Who teach because they love the teacher's task,
And find their richest prize
In eyes that open and minds that ask."
—*Henry Van Dyke*

• MEMORABILIA MATHEMATICA

Edited by William L. Schaaf, Department of Education, Brooklyn College, Brooklyn, New York

Mathematics and public relations

Strange indeed are the ways of public fancy. During the first quarter of the twentieth century, mathematics was held in unquestioned esteem—it was generally regarded as a highly respectable, if somewhat esoteric, subject. Yet within a few brief years it sank to the lower depths, if not literally, then figuratively. In any event, as a subject, or discipline, mathematics was *persona non grata* on two accounts: (1) professional mathematicians were hard put to it to justify their very existence, and (2) teachers of mathematics in the high schools and colleges found it increasingly more difficult to defend their subject in the eyes of the world.

Let us look first at the professional mathematicians' plight. Back in 1937, S. T. Sanders[1] stated the issue gently, but none the less firmly:

> It is questionable whether two chess players have a moral right to pursue their game while the neighbor's house burns down. If mathematical thinking is a pattern of true thinking, then may there not be a moral obligation resting on the mathematician to demonstrate this truth before the world so that the world may profit by its value, so that the vast potentials of mathematics may be applied to problems of crying human need as well as to those of science?

Two years earlier, E. T. Bell had put the case bluntly and even more forcibly:[2]

> For it must be obvious, even to a blind imbecile, that American mathematics and mathematicians are beginning to get their due share of those withering criticisms, motivated by a drastic revaluation of all our ideals and institutions, from the pursuit of truth for truth's sake to democratic government, which are only the first, mild zephyrs of the storm that is about to overwhelm us all. In the coming tempest only those things will be left standing that have something of demonstrable social importance to stand upon. Mathematics, we as mathematicians believe, has so much enduring worth to offer humanity on all sides from the severely practical to the ethereally cultural or spiritual, that we feel secure—until we stop to think.
>
> The arresting thought that we as mathematicians have done next to nothing to inform and convince the sweating men and the sweated women, whose hard labor makes possible our own leisurely pursuit of "the science divine," that mathematics does mean something in their lives and might mean more, may well make us apprehensive of the future, for these too patient men and women in the storm ahead of us all will cast the deciding vote.
>
> The harsh attrition has already begun. Are not mathematicians and teachers of mathematics in liberal America today facing the bitterest struggle for their continued existence in the history of our Republic? American mathematics is exactly where, by common social justice, it should be—in harnessed retreat, fighting a desperate rear guard action to ward off annihilation. Until something more substantial than has yet been exhibited, both practical and spiritual, is shown the non-mathematical public as a justification for its continued support of mathematics and mathematicians, both the subject and its cultivators will have only themselves to thank if our immediate successors exterminate both.
>
> Taking a realistic view of the facts, anyone but an indurated bigot must admit that mathematics has not yet made out a compelling case for democratic support, so that the men and women who pay the bills which make mathematics possible can see clearly what they are asked to pay for. This must be done, and immediately, if mathematics is to survive in America.

These were the years—the middle thirties and early forties—when mathematical education was severely under fire. Mathematics was condemned on every hand as too conservative, hide-bound, unrealistic, of no value, of no interest, too difficult, and so on *ad nauseam*. Then, too, it suffered through competition with other subjects, notably the social studies. The "mortality" was excessive. It was often

[1] Editorial, *National Mathematics Magazine*, February 1937.

[2] E. T. Bell, *The American Mathematical Monthly*, November 1935, p. 559.

damned by the grapevine before the teacher had a chance. But this is an old story which need not be repeated here.

With the advent of World War II there was a complete shift of scene. Mathematics was embraced as a most significant subject. One couldn't win a modern war without mathematics: the "men behind the men behind the guns" epitomized the pulse of public opinion.

But this meteoric blaze of popularity was short-lived. Hardly had the fighting stopped, when the slump in the favorable reputation of mathematics began to decline. Five years later, a writer[3] from "down under" could say with confidence: "Until very recently the supreme importance of mathematics was accepted; today its pretensions are hotly disputed." We venture the opinion that this is a fairly accurate appraisal (unfortunately) of mathematical education in America and on the Continent, just as surely as it is in Australia and New Zealand—certainly wherever some semblance of public, democratic mass education appears.

What of the future? What should be done about it? What *can* be done? On every hand we hear of the critical shortage of scientific and mathematical manpower. Every day the shortage of teachers is becoming more acute. Are we headed for the doldrums once more? The picture looks rather dark at the moment for mathematical education, although an occasional ray of hope pierces the gray where professional mathematicians and mathematical research are concerned. It is up to the young teachers of mathematics, and the mathematics teachers of tomorrow, to carry the torch. It is hard to believe that mathematics will long stay submerged—the subject of which G. H. Hardy so aptly has said:[4]

In these days of conflict between ancient and modern studies, there must surely be something to be said for a study which did not begin with Pythagoras, and will not end with Einstein, but is the oldest and the youngest of all.

[3] J. H. Murdoch, *The Teaching of Mathematics in Post-Primary Schools*, New Zealand Council for Educational Research, 1950, p. 154.

[4] G. H. Hardy, *A Mathematician's Apology*, (New York: Cambridge University Press), 1940, p. 16.

The Lobster Quadrille

Who does not recall with faint nostalgia the Mock Turtle as he sang the Lobster Quadrille to an enthralled Alice:

"Will you walk a little faster?" said a whiting to a snail,
"There's a porpoise close behind us, and he's treading on my tail" . . .

Which brings to mind the sprightly parody written some years ago by an esteemed friend and colleague, the late Professor Harris F. MacNeish, one-time Chairman of the Department of Mathematics at Brooklyn College. The amusing piece was originally published in the Brooklyn College *Math Mirror*, an annual publication of the College mathematics society, and, so far as we are aware, has not appeared elsewhere. It is reprinted here through the kindness of Miss Agnes MacNeish, a sister of Professor MacNeish.

The Hocus Pocus of the Locus

(Sung by Regulus Congruence Complex)
by Prof. Harris F. MacNeish

Converge a little faster,
  said the Limit to the Series,
There's a factor close behind us,
  and it hardly ever varies.
We are meeting at the focus,
  will you please describe the locus.
Will you, won't you, will you, won't you,
  will you please describe the locus;
Will you, won't you, will you, won't you,
  won't you please describe the locus.

You can really have no notion,
  how definite it will be,
When they take us up and count us,
  with the constants, 1, 2, 3.
But the Series answered sadly,
  "Sum me to infinity."
And thanked the Limit kindly,
  but he would not plot the locus.
Would not, could not, would not, could not,
  would not plot the locus;
Would not, could not, would not, could not,
  could not plot the locus.

What matters it what terms we take,
his constant friend replied,
There is another boundary, you know,
upon the other side.
The more you leave the vertex, dear,
the closer is the focus,
Then turn not pale, beloved scale,
but come and graph the locus.
Will you, won't you, will you, won't you,
will you graph the locus;
Will you, won't you, will you, won't you,
won't you graph the locus.

Life insurance in swaddling clothes

Virtually every mathematics curriculum and course of study today includes some reference to the basic ideas of life insurance —be it in eighth-year arithmetic, in the junior high school mathematics course, the general mathematics of the senior high school, or that more or less anomalous course known as "Consumers' Mathematics." All of which is mute testimony to the tacit conviction that it is important for the masses of our young people to know something about the mathematical ideas which underlie all insurance theory.

That the general dissemination of such knowledge is desirable is by no means a new idea. Let us read what the venerable Augustus de Morgan[5] had to say on the subject, somewhat over a hundred years ago, when life insurance was still in swaddling clothes.

> The theory of insurance, with its kindred science of annuities, deserves the attention of the academical bodies. Stripped of its technical terms and its commercial associations, it may be presented in a point of view which will give it strong moral claims to notice. Though based upon self-interest, yet it is the most enlightened and benevolent form which the projects of self-interest ever took. It is, in fact, in a limited sense, and a practicable method, the agreement of a community to consider the goods of its individual members as common. It is an agreement that those whose fortune it shall be to have more than average success, shall resign the overplus in favour of those who have less. And though, as yet, it has only been applied to the reparation of the evils arising from storm, fire, premature death, disease, and old age; yet there is no placing a limit to the extensions which its application might receive, if the public were fully aware of its principles, and of the safety with which they may be put in practice.
>
> It is of great importance at the present moment that sound principles on the subject of insurance should be widely and rapidly disseminated. Within the last twenty years, many new institutions have been founded; and during the busy portion of the London year, seldom a month passes without the announcement of a novel plan. Of many of these projects I am unable to speak, from not having paid particular attention to them. But of one thing I am certain, that the magnificent style in which the prospectuses frequently indulge might often remind their readers of the unparalleled benefits which are promised by another description of traders, who vie with each other in describing the rare qualities of their several blackings. If there be in this country a person whose ambition it is to walk in the brightest boots to the cheapest insurance office, he has my pity: for, grant that he is ever able to settle where to send his servant, and it remains as difficult a question to what quarter he shall turn his own steps. The matter would be of no great consequence if persons desiring to insure could be told at once to throw aside every prospectus which contains a puff: unfortunately this cannot be done, as there are offices which may be in many circumstances the most eligible, and which adopt this method of advertising their claims. If these pompous announcements be intended to profess that every subscriber shall receive more than he pays, their falsehood is as obvious as their meaning; if not, their meaning is altogether concealed.
>
> Public ignorance of the principles of insurance is the thing to which these advertisements appeal: when it shall come to be clearly understood that in every office some must pay more than they receive, in order that others may receive more than they pay, such attempts to persuade the public of a certainty of universal profit will entirely cease.

[5] *Essay on Probabilities, and Their Application to Life Contingencies and Insurance Offices* (London: The Cabinet Cyclopaedia, 1838), pp. XV–XVI.

On the nature and source of mathematical ideas

Where do mathematical ideas come from? Are they discoveries or inventions? To what extent are they shaped by external forces? What is the role of imagination? of sensory experiences? These and other questions beg to be answered. Writers like J. W. N. Sullivan, George Sarton, D. J. Struik, J. Hadamard, G. Polya and Max Wertheimer have at various times given us a meagre clue. Basi-

cally, however, these questions have not as yet been satisfactorily answered.

Meanwhile, a few pointed passages may well prove illuminating. Thus in Sarton's[6] opinion:

> There is no doubt that mathematical discoveries are conditioned by outside events of every kind, political, economic, scientific, military, and by the incessant demands of the arts of peace and war. . . .

Nevertheless, he continues:

> The main sources of mathematical invention seem to be within man rather than outside of him: his own inveterate and insatiable curiosity, his constant itching for intellectual adventure; and likewise the main obstacles to mathematical progress seem to be also within himself; his scandalous inertia and laziness, his fear of adventure, his need of conformity to old standards, and his obsession by mathematical ghosts.

He observes further:

> The deterministic theory of mathematical progress remains insufficient even when one has corrected and tempered it. . . . Many mathematical developments are capricious in the extreme, and it is a waste of time to try to find a rational explanation of them. . . . The capriciousness of mathematical development cannot be emphasized too much.

Perhaps a more pregnant clue lies in the relation of mathematics to science. We should like therefore to present three interpretations given by quite different personalities. The first may be familiar to our readers, but the other two are not likely to be. We think that these three passages in juxtaposition may prove rather suggestive. More than a quarter of a century ago, J. W. N. Sullivan wrote:[7]

> Since, then, mathematics is an entirely free activity, unconditioned by the external world, it is more just to call it an art than a science. . . . The significance of mathematics lies precisely in the fact that it is an art; by informing us of the nature of our own minds it informs us of much that depends on our minds. It does not enable us to explore some remote region of the externally existent; it helps us to show us how far what exists depends upon the way we exist. We are the law-givers of the Universe; it is even possible that we can experience nothing but what we have created, and that the greatest of our mathematical creations is the material universe itself.
>
> We return thus to a sort of inverted Pythagorean outlook. Mathematics is of profound significance in the universe, not because it exhibits principles that we obey, but because it exhibits principles that we impose.

The next passage is from the pen of a well-known person, neither a mathematician nor an historian. According to Alexis Carrel:[8]

> . . . we love to discover in the cosmos the geometrical forms that exist in the depths of our consciousness. The exactitude of the proportions of our monuments and the precision of our machines express a fundamental character of our mind. Geometry does not exist in the earthly world. It has originated in ourselves. The methods of nature are never so precise as those of man. We do not find in the universe the clearness and accuracy of our thought. We attempt, therefore, to abstract from the complexity of phenomena some simple systems whose components bear to one another certain relations susceptible of being described mathematically.

And finally, the physicist[9] speaks:

> On the one hand, mathematics is a study of certain aspects of the human thinking process; on the other hand, when we make ourselves master of a physical situation, we so arrange the data as to conform to the demands of our thinking process. It would seem probable, therefore, that merely in arranging the subject in a form suitable for discussion we have already introduced the mathematics—the mathematics is unavoidably introduced by our treatment, and it is inevitable that mathematical principles appear to rule nature.

Yet even with such light as has been shed upon these questions, we still do not know whether Descartes was right when he insisted that "a triangle does not depend in any way upon one's mind; it has an eternal and immutable existence quite independent of our knowledge of it." Or, with Hermite and Hadamard: " 'We are rather servants than masters in Mathematics.' Although the truth is not yet known to us, it preexists and inescapably imposes on us the path we must follow under penalty of going astray."

[6] George Sarton, *The Study of the History of Mathematics* (Cambridge, Mass.: Harvard University Press, 1936), p. 16 ff.

[7] J. W. N. Sullivan, *Aspects of Science: Second Series* (New York: Alfred Knopf, 1926), pp. 93 ff.

[8] Alexis Carrel, *Man the Unknown* (New York: Harper and Bros., 1935), p. 8.

[9] W. Heisenberg, as quoted by P. W. Bridgman in *Science*, 1930, vol. 71, p. 21.

Reviews and evaluations

Edited by Richard D. Crumley, University of South Carolina, Columbia, South Carolina, and Roderick C. McLennan, Arlington Heights High School, Arlington Heights, Illinois

BOOKS

Mathematical Puzzles and Pastimes, Aaron Bakst, New York, D. Van Nostrand, Inc., 1954. vii+206 pp., $3.75.

Any book on mathematical recreations will find enthusiastic reception from both teacher and student. Some teachers, however, have complained that most such books are merely collections of "trick" problems, with little attention to mathematical principles. Definitely this book is an answer to such criticism.

Although much of the material is not new, the well-organized, systematical treatment is unusual. The reader can not only enjoy a recreational problem, he is shown some of the underlying theory, and should be able to systematically attack problems of this type, and to construct similar problems of his own.

It is hard to single out any particular section, but this reviewer found especially interesting the material on a "billiard ball computer" to solve container problems and the fine treatment of systems of numeration (here the reader not only finds recreational material, but also should gain a good understanding of various number systems). Students will be intrigued to see how we might restore our knowledge of trigonometry, should some catastrophe wipe out all mathematics books.

Certainly this book should be in every high school or college library; the teacher would do well to have a personal copy. Not only is the treatment interesting, it is mathematically sound.—*Cecil B. Reed, University of Wichita, Wichita, Kansas.*

Arithmetic for High Schools, Charles H. Butler, Boston, D. C. Heath and Company, 1953. Cloth, xv+336 pp., $2.40.

In his book, Mr. Butler has done an excellent job of coordinating skill and understanding in arithmetic. The illustrations attract attention because of their clarity and appropriateness. The problems are varied and practical. The testing program deserves favorable comment.

Perhaps somewhere in the chapter on "Squares and Square Roots" the exact method for finding square roots by computation might be explained. In the chapter "Some Common Geometric Forms," should there not be some explanation as to "how" to construct as well as the pictures of constructions? Other than the above-mentioned incidents, I think the book is an excellent one for high school use.—*Marie Swan, Monroe High School, St. Paul, Minnesota.*

Elements of Number Theory, by I. M. Vinogradov, translated from the Fifth Revised Edition by Saul Kravetz, New York, Dover Publications, Inc., 1954. Cloth, viii+227 pp., $3.00; Paper, $1.75.

This is the first English translation of the fifth Russian edition of 1949. This fifth edition differs considerably from the fourth. The most important changes are the merging of the old chapters IV and V into one chapter IV (reducing the number of chapters to six) and the new, simpler proof of the existence of primitive roots.

The material for the book is well chosen. The contents include divisibility theory, important number-theoretical functions, congruences, congruences in one unknown, congruences of second degree, primitive roots and indices. Also, the book contains tables of indices and tables of primes <4000 and their least primitive roots.

The 233 problems and their solutions and also the 104 numerical exercises and their answers are exceptionally good. Some of these deal with the estimation of trigonometric sums which serve as an important introduction to Vinogradov's methods for solving the problems of Waring and Goldbach. However, from the viewpoint of the teacher the reviewer questions the advisability of including the solutions to the problems within the book.

It is stated in the book that the reader who has not progressed beyond college algebra would have no trouble following the discussion and working out 95% of the problems. It seems to the reviewer that this statement is somewhat misleading for it would require considerable mathematical maturity to understand all the theory and to work most of the problems. Some of the notation is introduced without an explanation and the problems extend the theory considerably.

This book fills a need for a beginning course in number theory.—*Clyde T. McCormick, Illinois State Normal University, Normal, Illinois.*

INSTRUMENT

Vernier Calipers ($3.25)

Yoder Instruments, East Palestine, Ohio. Caliper rule with sliding indicator; inside and outside capacity 5″; $1.50.

This all-metal caliper rule has two scales—one using inches with one-sixteenth subdivisions, the other using centimeters with millimeter subdivisions. The sliding mechanism incorporates one jaw for finding outside dimensions, one jaw for inside dimensions, and the two vernier scales for inches and centimeters. The sliding mechanism can be locked at any position by tightening a knurled knob. A small knurled roller is attached to the sliding mechanism to assist in making small movements. The vernier scales make it possible to find dimensions to the nearest one-hundred-twenty-eighth of an inch or to the nearest hundredth of a centimeter. An additional feature of the rule is a thin metal strip ($\frac{1}{4}$″ wide) which is attached to the sliding mechanism. This strip protrudes from one end of the rule enabling one to measure the depth of holes.

This rule is exceptionally good considering the price of it. It is not as precise as the regular tool maker's calipers, yet it can serve well in mathematics classes. Several of these calipers could be used by students in a secondary school mathematics class to acquire an understanding of vernier scales and the metric system of linear measure, or they could be used in classroom activities requiring accurate measurement.

—*Richard D. Crumley*

MODEL KIT

Scientific Makit

W. R. Benjamin Company, Granite City, Illinois. Construction set of 48 wooden balls, 16 wooden wheels, 192 wooden rods, and accessory parts; packed in metal box with hinged tray; postage prepaid; $5.00 or $5.50 west of Denver.

This set is similar to some toy sets except that more balls and long rods are included in the set. The balls and wheels have diameters of $1\frac{3}{4}$″, and the rods have lengths of $1\frac{3}{4}$″, $2\frac{1}{2}$″, $3\frac{1}{2}$″, $5\frac{1}{4}$″, $7\frac{3}{4}$″, and $11\frac{1}{2}$″. The wheels are colored red, blue, or yellow, and the balls are colored red, blue, yellow, orange, brown, or white. The rods fit snugly into any of the seventeen holes in a ball or any of the ten holes in the wheels. (There are eight white balls with only two holes in each ball.) There are also several red rods (slightly thinner than the other rods) which slide easily through the holes in the wheels and balls. The set is packed in a metal box similar to a fishing tackle box.

This set can be used effectively in grades seven and eight but especially in a solid geometry class. The rods and balls can be put together to form the edges of various solid geometric figures. The motivation resulting from students "doing it themselves" should enhance the usefulness of this set. The ability to get inside a solid model constructed with the set is another asset. Planes made of poster board could also be used nicely with the set.—*R. D. C.*

Have you read?

Gallacher, Ted and Stevens, Ray C. "Teaching With a Tape Recorder," *California Journal of Secondary Education*, Vol. 29, No. 1, October 1954, pp. 312–314.

With overflowing classrooms the situation today, all teachers face the almost impossible task of making provisions for recognizing individual differences. This article describes how one teacher partially solved the problem by enlisting the aid of a tape recorder.

Six sets of headphones and several tapes on which suitable material had previously been recorded constituted the total equipment. You will want to read how this method worked for teaching the multiplication facts to those students of lower ability. The authors also point out possibilities of using the tape for historical facts, material for superior students, practice material in all areas and in providing a much broader program of learning. You will also be interested in teacher reaction in general.

Jacobs, Leland B. "What about Home Work?" *Childhood Education*, Vol. 31, No. 2, October 1954.

Although this is a very old subject and there have been volumes written on it, I believe it will still be of interest to you. The approach is different and the analysis of the purposes is also different.

You will be interested to note why homework and school work are not separate learning activities, and how the teacher and the home must understand the procedures and purposes. You will be pleased with the idea of homework as creative work rather than practice; that homework places the student in the position of a chosen contributor, of a responsible member of the group; and he becomes a necessary part of the total organization. The suggestions for homework are good and you will probably want to add many more.—*Philip Peak, Indiana University, Bloomington, Indiana.*

• TIPS FOR BEGINNERS

Edited by Dr. Francis G. Lankford, Department of Education, University of Virginia, Charlottesville, Virginia

Elementary concepts of irrational numbers

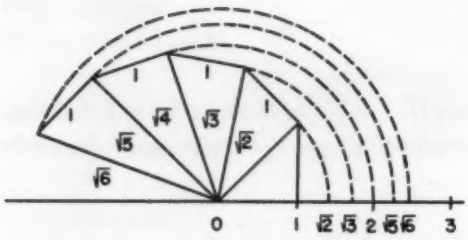
by Malcolm Graham, Longwood College, Farmville, Virginia

Integers, fractions, and decimal fractions are used so extensively in measuring such things as distances, volumes, forces, populations, weights, and wealth that they fall within the realistic experience of nearly everyone. Since all physical measurements are approximate and since any degree of precision can be indicated by the rational numbers, these are the numbers which are used in graduating scales, in indicating bank balances, and in expressing velocities and accelerations. Through their usage the rational numbers have real meaning for most individuals.

Probably no high school student will state that ninety-nine hundredths is not a number since there is no integer to which it is exactly equivalent. But many high school students will claim that the square root of two is not a number because there is no fraction or decimal fraction to which it is equal. In the thinking of many students, irrational numbers are classified along with such things as perpetual-motion machines, perfect reflectors, and thermos bottles which keep coffee hot indefinitely. To this group of students the things which irrational numbers represent exist only in the imagination and the ideals which they represent can be indicated only in an approximate manner by annexing more and more digits to decimal fractions. These students would agree that a rubber band could theoretically be stretched to a length of exactly 7 inches, but they would be exceedingly skeptical concerning the possibility of its being stretched to a length of exactly $\sqrt{50}$ inches.

What can be done to help these students attain a better concept of irrational numbers? A knowledge of how irrational numbers can be used in representing such things as lengths, forces, and velocities is invaluable in convincing students that the numbers $\sqrt{2}$, $\sqrt{7}$, log 3, and sin 23° are just as "real" as the rational numbers.

Before studying irrational numbers, however, the student must first understand the meaning of rational numbers and realize that any rational number can be represented as a fraction a/b where a and b are integers. The geometric concept of a fraction is easily developed with the aid of the number scale. It is readily seen that every fraction can be located as a point on the number scale—those with denominators of two by bisecting each unit, those with denominators of three by trisecting each unit, and so on. It is important to note that any fraction may represent not only a point on the number scale but also the distance from zero to that point. Many students would suppose that, if all possible fractions were represented by points on the number scale, the scale would be filled in solidly. It is true that the points would be dense but they would not form a continuous succession. As a matter of fact, more points would be omitted than would be included. Before demonstrating that many numbers such as $\sqrt{2}$, π, and log 3



would be omitted, it is helpful to show how a few of the irrational numbers can be used in representing geometric distances. For example, after choosing any arbitrary distance as unity, the Pythagorean theorem may be applied in representing geometrically the square roots of all integers. Now represent these distances on the number scale as in the figure and challenge the student to find a fraction a/b which used as a factor twice will equal 2. If there is such a fraction, then $\sqrt{2}$ is rational. Of course, the student will be unable to find such a number, since all fractions are either larger or smaller than $\sqrt{2}$. Let the student attempt in a similar manner to find $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, and $\sqrt{6}$. He will soon sense that $\sqrt{N}$, where N is an integer, is rational only when N is the square of an integer as is the case of 4, 9, 16, 36, and so on.

After this preliminary work the student should be in a position to appreciate some easy proofs in which it is shown that certain selected numbers are not rational. Two such proofs follow.

Proof that $\log_{10} 5 \neq a/b$

If $\log_{10} 5 = a/b$, (a and b, integers) then $10^{a/b} = 5$ and $10^a = 5^b$. But the last digit of 10^a is always zero and the last digit of 5^b is always five. Therefore, 10^a cannot equal 5^b and our original supposition that $\log_{10} 5 = a/b$ is false.

Proof that $\sqrt{2} \neq a/b$ (a and b, integers)

Assume $\sqrt{2} = a/b$ and that a and b are not both divisible by the same integer (other than 1); that is, a and b are relatively prime. Then $2 = a^2/b^2$ and $2b^2 = a^2$. Since $2b^2$ is divisible by 2 it is an even number; therefore a^2 is even. But if a^2 is even a must also be even since the square of an odd number is always odd. Since a is even, let $a = 2k$. Then, by substitution, $2b^2 = (2k)^2$ or $2b^2 = 4k^2$. Thus, $b^2 = 2k^2$. Since $2k^2$ is even, b^2 is even and b must also be even. It has now been shown that both a and b are even and therefore have a common factor of 2. Hence our original hypothesis that a and b are relatively prime is contradicted and this inconsistency leads us to conclude that $\sqrt{2} \neq a/b$.

A note on the statements of theorems and assumptions

by Charles H. Butler, Western Michigan College of Education, Kalamazoo, Michigan

Sometimes proofs of theorems contain statements which appear to high school students to be nonsensical or untrue. This is especially likely to happen in indirect proofs. For example, consider the following rather typical presentation of a theorem in plane geometry.

"Given: PQ and RS cut by AB with angle x = angle y.

"Prove: $RS \parallel PQ$.

"Proof: Assume that angle x = angle y, . . . etc." The argument follows, and ends with the conclusion (logically correct but inconsistent with the appearance of the diagram used in the proof) that $RS \parallel PQ$.

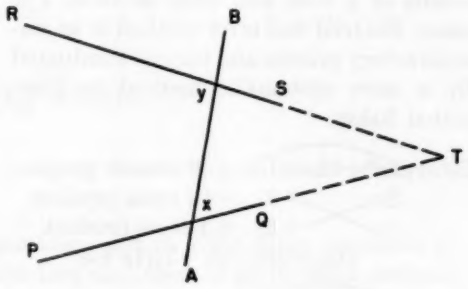
Such apparent discrepancies between the appearance of a diagram and the stated assumptions and conclusions will not bother mature mathematicians, but they do bother immature high school students

who are just learning what a demonstration means. To them it doesn't make sense to say "assume that $\angle x = \angle y$" when they can see from the diagram that angle x actually is not equal to angle y, or to conclude that $RS \parallel PQ$ when these lines can be seen actually to intersect in the diagram.

On the other hand, most students, even though mathematically immature, would find no difficulty in accepting the statement that "IF angle x WERE equal to angle y, then RS WOULD BE parallel to PQ" as a reasonable statement even though the diagram does not appear to exhibit these conditions.

To cite a non-geometrical proposition which is the exact counterpart of the foregoing example, it might easily appear ridiculous to young students for a 5½ foot man to say, "Assume that I am 9 feet tall; therefore I can touch the ceiling," but the same students would readily accept from the same man the statement, "IF I WERE 9 feet tall, THEN I COULD touch the ceiling" and regard it as a perfectly normal kind of statement.

Logically there is no difference between saying, "Assume this to be true; therefore that is true" and "If this were true, then that would be true," but there is a profound difference in the attitude or feeling induced in immature students by the two forms of statement. Surprising improvement in interest and in understanding of the meaning and nature of demonstration can often be brought about by this simple change in the wording of key statements.



Have you read?

LE CORBEILLER, P. "The Curvature of Space," *Scientific American*, Vol. 191, No. 5, November 1954, pp. 80–86.

Here is an article you cannot afford to have your students of plane geometry miss. It brings to life the thinking and the problems around the origin of non-Euclidean geometry. It will bring to any student of geometry a broader outlook and a greater appreciation for this intriguing subject.

The subject comes to life when the student discovers that only a little over 100 years ago Bernhard Rieman gave his famous paper on foundations in geometry, which in turn opened the road for Einstein's Theory of Relativity. The story becomes very human when one reads what Rieman wrote to his father after the great mathematician, Gauss, requested Rieman's presentation of the foundations paper before he would be considered as a candidate for lecturer in the University. I am sure they will also be interested in the discussion of different surfaces such as the egg and the saddle. The very best students in your classes will analyze the equations given and will also realize Rieman was not talking in the abstract but realized the physical possibilities which time has borne out. I am sure you will enjoy this article also.

SYER, HENRY W. "Making Mathematics Sensible," *N.E.A. Journal*, Vol. 43, No. 4, April 1954, pp. 221–223.

Henry Syer, whose evaluations of teaching materials you have been reading in THE MATHEMATICS TEACHER, has in this article drawn together some very good thoughts on all the sources of materials and to which of our senses they apply in making mathematics more sensible and good mathematics teaching better. Believing in what he writes, Mr. Syer has set up a very impressive chart showing this. You will also profit from the suggestions given on how to use each of these and the methods of making or obtaining many different aids. Although this is a very short article, it is packed with ideas. Don't miss it.—*Philip Peak, Indiana University, Bloomington, Indiana.*

Edited by John A. Brown, University of Wisconsin, Madison 6, Wisconsin, and Houston T. Karnes, Louisiana State University, Baton Rouge 3, Louisiana

Deciphering the general trinomial

by Gifford Welling, Greenwich Central School, Greenwich, New York

Sometimes teachers teach the factoring of the general trinomial ax^2+bx+c by means of a trial and error method. For some, the trial and error method is an unsatisfactory process and may be eliminated by a more systematic method as illustrated below.

Ex. 1: $2x^2+13x+15$, +30 master product
$2x$ $+3$, +10 cross product
$1x$ $+5$, +3 cross product
therefore: $(2x+3)(1x+5)$

Ex. 2: $3x^2-13x-10$, −30 master product
$3x$ $+2$, −15 cross product
$1x$ -5, +2 cross product
therefore: $(3x+2)(1x-5)$

Ex. 3: $4x^2-11x+6$, +24 master product
$4x$ -3, −8 cross product
$1x$ -2, −3 cross product
therefore: $(4x-3)(1x-2)$

Ex. 4: $6x^2+7x-3$, −18 master product
$3x$ -1, +9 cross product
$2x$ $+3$, −2 cross product
therefore: $(3x-1)(2x+3)$

Ex. 5: $12x^2+17x+6$, +72 master product
$4x$ $+3$, +9 cross product
$3x$ $+2$, +8 cross product
therefore: $(4x+3)(3x+2)$

Ex. 6: $12x^2-x-6$, −72 master product
$4x$ -3, −9 cross product
$3x$ $+2$, +8 cross product
therefore: $(4x-3)(3x+2)$

Let us consider *how* to work Example 1 above. Simply multiply $2\times15=30$, "the master product." Determine the cross products, +10 and +3, in exactly the same way that a student would factor $x^2+13x+30$. Divide 2 into the only cross product into which it will divide; namely 10, obtaining 5. Divide 1 into the remaining cross product; namely 3, obtaining 3. This sounds very complicated in paragraph form, but it is very easy to do.
First stage:

$2x^2+13x+15$, +30 master product
$2x$ $+10$ cross product
$1x$ $+3$ cross product

Second stage:

$2x$ $+3$
$1x$ $+5$
therefore: $(2x+3)(1x+5)$

The sign difficulties in Examples 2, 3, and 4 take care of themselves automatically.

The only remaining difficulty is handled as follows: in Example 3, the 4 cannot be factored into 2 and 2 since *one* of the 2's cannot be divided into 3, therefore one must use 4 and 1.

In Example 4, the 6 cannot be factored into 6 and 1 since 6 cannot divide either 9 or 2. Therefore one must use 3 and 2.

In Example 5, the 12 cannot be factored into 12 and 1 since 12 cannot divide either 9 or 8. 12 cannot be factored into 6 and 2 since 6 cannot divide either 9 or 8. Therefore one must use 4 and 3.

This article sounds complicated in paragraph form, but once it is understood by the teacher, it is very easy to teach and very easy for the student to understand.

Mathematics is one subject that a high school freshman cannot learn in paragraph form from a textbook. Good blackboard teaching, accompanied by oral explanation, is indispensable in mathematics.

Finally, let me make it clear that it is not necessary, and probably not advisable, to teach any of the following complicated paragraphs to students. This is written for teacher consumption to establish the validity of the method and to show why it works. Considering Example 1 above, the validity of this method is based upon the fact that the product of 2, 1, 3, and 5 in any order always gives the same master product, namely 30. But $2\times1=a$ which is 2 and $3\times5=c$ which is 15. Therefore $a\times c$ (which happens to be 2×15 in this case) does give the master product, namely 30. But $2\times5=10$, one cross product and $1\times3=3$, the other cross product. Naturally the two cross products must multiply to get the same master product, namely 30, since they contain the products of the same four numbers. In other words, the product of the vertical products must give the same master product as the product of the cross products. a and c are always vertical products, 2 and 15 in this case.

The builder of an enquiring mind[1]

by E. G. MacKenzie, Denver, Colorado

There is one facet of learning open to all energetic teachers of mathematics. This facet will produce the typically spirited remarks: "I didn't know geometry had anything to do with algebra. You sure have to know a lot of mathematics to work these problems." Excluding the regular formal classroom situation, one activity to bring together the elements of physics, chemistry, and mathematics, and to demonstrate forcefully the interrelationship of these subjects is the mathematics club.

This is one organization where time can be allotted to create in the mind of each club member a sense of appreciation for engineering and scientific applications of mathematical facts; for re-emphasizing fundamental concepts that up to this time have had little meaning to the individual; and for instilling a feeling that will impel him to investigate unique applications whenever or wherever they arise.

When planning and carrying out various club activities, it is necessary to have complete co-operation among the club members and to make them cognizant of the fact that there is no imposed demand for learning, but that, through interest, various problems are attacked and solved. Because of the lack of students' experiences and mathematical background, the sponsor may need to introduce and initiate discussion of new material. However, all activities should be put into the hands of the members whenever possible. Frequently, students are capable of doing the necessary research on a problem to be discussed and of leading the discussion. Yet at all times the sponsor must be alert and ready to assist tactfully in clarifying vague ideas.

A good lead-off activity for the fall semester is the slide rule. The student planning toward a career as a scientist or an engineer should not be without a truly proficient understanding of this tool of mathematics. A sponsor with an appreciation for applications can insure that all the fundamentals of the slide rule are thoroughly mastered.

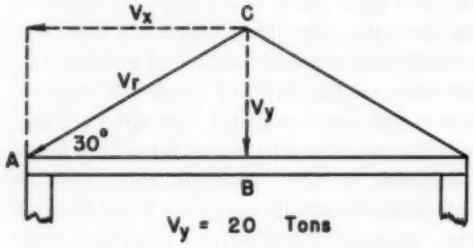
Often simple scientific and engineering applications are dealt with ineffectively in the classroom because of the limited time

[1] Reprinted from the April 1954 issue of *The Encompasser*, Colorado Council of Teachers of Mathematics.

and background of the students. But it is just these applications that give impetus to the students' realization that those seemingly unrelated facts previously learned in arithmetic, geometry, and algebra must be used together to solve problems. Several applications, which can be used in the mathematics club, are noted for your consideration.

In geometry, how many of us have recently thought of the practical applications of the 30° angle and Pythagorean theorems, or the trigonometric solutions to a bridge construction problem involving the resolution of vector forces.

Problem: Knowing that the force acting downward at the junction of the girders is 20 tons, what will be the stress exerted upon the horizontal member of the bridge? What will be the resultant force exerted by the girder forming a 30° angle with the horizontal member of the bridge?



V_r—resultant vector force
V_x—horizontal component force
V_y—vertical component force

Solution 1 (30° angle and Pythagorean theorem)

Resolving V_r into V_x and V_y by the rectangular method, V_x represents the stress on the horizontal member of the bridge—the opposite sides of a rectangle are equal. Noting that V_y is opposite the 30° angle, V_r will be 40 tons—the side opposite the 30° angle is equal to one-half the hypotenuse in the right triangle formed. Applying the Pythagorean theorem

$$V_x = \sqrt{V_r^2 - V_y^2} = \sqrt{1600 - 400}$$

$V_x = 34.6^+$ tons (slide rule solution)

Solution 2 (Algebra, 30° angle and Pythagorean theorems)

Let $V_x = \sqrt{3X}$ and $V_r = 2X$

Then $(2X)^2 = (\sqrt{3X})^2 + (20)^2$, etc.

Solution 3 (Trigonometry and 30° angle theorem)

$V_r = 40$ tons, then $V_x = 40$ Cos 30°, etc. (slide rule solution).

A group of problems illustrating applied mathematics is found in the article, "Some classroom problems from the field of atomic energy" by Wallace Manheimer.[2] The major ideas of this article are: (1) representation of numbers in exponential form, demonstrating the conversion of matter to equivalent energy; (2) the use of geometric progressions and logarithms to determine the life span of radioactive elements and the multiplication factor of an atomic reactor; (3) the use of ratio and proportion to determine the critical size of a reactor or the atomic bomb, the separation of isotopes, and the gaseous diffusion of molecules; (4) derivation of formulas.

The geometric series, cited above, presents a fine opportunity to re-emphasize the fundamentals of arithmetic, algebra, and theory of limits. By illustrating the geometric series in terms of arithmetic—

$$S_n = 1(\tfrac{1}{2})^0 + 1(\tfrac{1}{2})^1 + 1(\tfrac{1}{2})^2 + 1(\tfrac{1}{2})^3 + \cdots + 1(\tfrac{1}{2})^{n-2} + 1(\tfrac{1}{2})^{n-1} + 1(\tfrac{1}{2})^n.$$

(n, a given number, produces a mixed number smaller than the largest possible.)

$$S_\infty = 1(\tfrac{1}{2})^0 + 1(\tfrac{1}{2})^1 + 1(\tfrac{1}{2})^2 + 1(\tfrac{1}{2})^3 + \cdots + 1(\tfrac{1}{2})^\infty$$

(produces the largest possible mixed number)—and then actually deriving the formulas for the sum of a geometrical series and the infinite geometric series, the student is shown that ultimately the fraction approaches zero as the denominator of the fraction becomes infinitely large. Here

[2] Wallace Manheimer, "Some classroom problems from the field of atomic energy," The Mathematics Teacher, XLVII, No. 2, February 1954, pp. 86–90.

meaning has been given to fundamental concepts of mathematics.

Also from the above cited article is the problem of separation of isotopes.

Problem: How many operations must be performed to double the concentration of Uranium 235 in a given process?

Separation factor $= F_1/F_2$, F_1 and F_2 being the concentration before and after the operation respectively. The separation factor for U-235 is 1.004.

$$(1.004)^n = 2$$
$$n \log 1.004 = \log 2$$
$$n = \frac{\log 2}{\log 1.004}, \text{ etc.}$$

Sources of material that provide ample problems and applications are found in any high school or college text on physics; in slide rule manuals, published by the manufacturers; and in THE MATHEMATICS TEACHER.[3] Certainly, there is a wealth of material to be used in our mathematics clubs. And it is the use of such material together with a well-planned program that will produce the desired effect—students developing an inquiring mind.

[3] Robert Schilling, "Mathematics in Engineering," THE MATHEMATICS TEACHER, XLVI, No. 1, January 1953, pp. 14–17.

Walter J. Seeley, "The Engineer's Simple Arithmetic," THE MATHEMATICS TEACHER, XLVI, No. 8, December 1953, pp. 565–566.

John Viall, "Mathematics of the Air-Craft Industry," THE MATHEMATICS TEACHER, XLVI, No. 3, March 1953, pp. 145–151.

High school teacher fellowship program

Continued from page 69

for a fellowship is limited to teachers (1) who have taught at least three years and have devoted at least half the time to classroom teaching in each of the past three academic years, and (2) who will not have reached their fiftieth birthday by January 1, 1955. Previous recipients of Fund Fellowships are not eligible to apply.

Forms for individual applicants, school superintendents and for local nominating committees are being distributed to superintendents in all high school districts throughout the country. A limited number of additional forms may be obtained from the Committee.

Individual applicants should not apply to the Fund for the Advancement of Education, but only to their superintendent of schools or local nominating committee.

The amount of the fellowship award will take into account the regular salary the teacher would receive during the school year (excluding summer, night school, or other "extra" work), but not less than $3,000, and reasonable allotments for other expenses. Only costs of transportation within the United States may be covered by the grant, though a recipient may undertake foreign travel at his own expense.

Recommendations of the local committees must be mailed so as to reach the offices of the National Committee on High School Teacher Fellowships *no later than March 15, 1955*. Final announcement of all fellowship awards will be made on or about April 20, 1955. All inquiries or other communications concerning the program should be addressed to the National Committee on High School Teacher Fellowships, The Fund for the Advancement of Education, 655 Madison Avenue, New York 21, New York.

THE COMMITTEE:

Mr. Omer Carmichael
Superintendent of Schools
Louisville, Kentucky

Mr. Herbert Clish
Superintendent of Schools
San Francisco, California

Mr. William H. Cornog
President, Central High School
Philadelphia, Pennsylvania

Miss Marcia Edwards
Associate Dean, College of Education
University of Minnesota

Mr. Harold Gores
Superintendent of Schools
Newton, Massachusetts

Mrs. Hortense Levisohn
Principal, William Howard Taft High School
New York, New York

Mr. Lloyd Michael
Superintendent, Evanston Township High School
Evanston, Illinois

Mr. William L. Pressly
President, The Westminster Schools
Atlanta, Georgia

Mr. Gerhardt E. Rast
Superintendent of Schools
Westport, Connecticut

Mr. Virgil Rogers
Dean, School of Education
Syracuse University

A column of unofficial comment

A challenge to educators

by Henry Van Engen, Iowa State Teachers College, Cedar Falls, Iowa

In a front page article in the *New York Sunday Times* for November 7, 1954, Benjamin Fine reports on the results of a *Times* study which compared the output of Russian scientists and engineers with that of the United States. The heading for the articles tells the story in a most striking manner. RUSSIA IS OVERTAKING UNITED STATES IN TRAINING OF TECHNICIANS is the line used to attract the attention of the reader to a full page report on the subject. This study needs serious consideration by all teachers of mathematics and educators in general. Here are some of the salient features of the study as gleaned from the *Times* article.

"While the democracies of the world, including the United States, are looking the other way, the Soviet Union and its satellites are training scientists and engineers at an almost feverish pace," says the *Times* article. Despite this effort by the Russians and the urgent need for scientists in the United States the supply in the United States has gone down in the last few years. The peak year of 1950 shows the colleges in America graduating 50,000 engineers. In the last year only 20,000 engineers graduated to enter the industrial world. In contrast, the Soviet Union graduated 40,000 engineers in 1953 and 54,000 in 1954. "At the present time, the Soviet Union is graduating two and one-half times as many engineers each year as is the United States."

After noting that a similar emphasis is being put on the training of scientists and engineers in satellite countries the *Times* article goes on to discuss the shortage of teachers of science and mathematics in the United States. The article gives the following table of figures which shows the number of graduates prepared to teach various subjects in the high school.

	Mathematics	*General Science*	*Chemistry*	*Physics*
1950	4,618	3,009	1,660	954
1951	4,118	2,772	1,342	578
1952	3,142	2,216	842	373
1953	2,573	1,664	662	357
1954	2,281	1,505	608	259

"This means, realistically, that the total number of college graduates who are prepared to teach high school science has dropped from 9,096 in 1950 to 3,978 in 1954, or a decrease of 56 per cent."

In comparing the status of science and mathematics in the Soviet schools and in the United States Mr. Fine says, "Science in the Soviet schools is stressed from elementary grades upwards. One-third of the seven-year elementary school curriculum consists of arithmetic, algebra, geometry, the natural science, physics and chemistry. Then in the secondary schools, 40 per cent of the curriculum is devoted to science and mathematics (there are no electives). . . . This is not done by chance. The Soviet Union has deliberately set out to take the lead in scientific and engineering fields."

What are some of the basic issues in this situation? Mr. Fine quotes Dr. T. H. Chilton, chairman of the Engineering Manpower Commission of the Engineers Joint Council, as saying, " . . . the problem of greatest concern to the engineering and

scientific professions at present is that of the secondary schools. Scientists are worried about the quality and quantity of mathematical and scientific instruction that the high school students receive. Under present conditions, instruction must be intrusted to teachers who have had little or no training in mathematics and science."

These highlights of the *Times* study should furnish food for thought for secondary teachers of mathematics. It behooves each teacher to consider what he can do as an individual to meet the problem so forcefully set forth in the *Times* study. While individual efforts seem puny when compared with the whole problem, one must remember that the problem can only be solved by individuals working in unison. However, if teachers would generally make more effective use of guidance materials, in an effort to interest high school students in the field of science, the problem would be partially solved. More high school students who have the ability must be encouraged to go to college.

The reading of the *Times* article does bring into focus one of the more pressing problems faced by educators interested in the high schools. Our curriculum in mathematics is essentially the curriculum of our fathers and grandfathers. Mathematics has changed since the time our grandfathers graduated from high school, but has the high school mathematics (and the undergraduate college mathematics, as well) made a corresponding change? There is much reason to believe that it has not. If we are to train more and better scientists then the present high school curriculum needs extensive modification. We cannot go on wasting our time on such courses as solid geometry, computational trigonometry and one whole year of plane geometry, and meet the problem squarely. The time is ripe for a change and the change must come.

Some reflections on a hurried trip to Europe

by Myron F. Rosskopf, Teachers College, Columbia University, New York

After having spent twenty days in five different European countries and having attended the International Congress of Mathematicians at Amsterdam, Mrs. Rosskopf and I feel like the reporter whose views on the European economic situation appeared recently in a financial weekly. At the beginning and the end of his article he apologized for presuming, on so short a visit as his, to have worth-while reflections. Nevertheless he wrote a rather long and careful analysis. So, too, I shall write a thoughtful analysis but not a long one.

One of the first things that impresses you as you talk with representatives from other countries about the teaching of mathematics is their wide knowledge of the whole area. Not only do these people know what is being done in their own countries, but they are familiar with the principal publications, books, and periodicals of other countries. For example, a young Norwegian mathematician had in his briefcase several recent copies of THE MATHEMATICS TEACHER that he had brought along with him to read during free moments at the Congress. Such experiences impressed upon me the necessity for our own leading mathematics educators to make great efforts to extend their knowledge beyond the limits of continental United States. Certainly mathematics educators in the United States are making contributions to better programs in mathe-

matics, but Europeans also are making contributions. We in this country should know about them.

Compulsory education for children in most countries extends through age fourteen. There is a high regard for the importance of schools. Some of the first buildings erected in bombed out cities of Germany after the war were elementary school buildings. Naturally, European countries face in education many of the problems that we face. However, their attempted solutions are different. The problem of differences in ability and objective is met by having several types of schools rather than by having several programs in the same school, particularly at the secondary school level.

At sessions of Section VII, on philosophy, history, and education, there were presented a series of papers on trends in mathematics education for students from age sixteen to twenty. It was apparent from these reports that the mathematics curriculum in the United States and the organization of the courses is quite different from any that exists in Europe. Part of the difference is due to a difference in philosophy concerning education. We try to carry as many students as we can and as far as possible into mathematics. The Europeans try to separate by application of rigorous standards those who can do mathematics from those who cannot. Both Howard Fehr and I noticed this difference in point of view. In addition, both of us sensed a feeling of questioning concerning their mathematics programs, at least in Norway, Denmark, and The Netherlands. To us it was interesting that in these countries, too, there is a struggle over what sort of educational program is better for young people. Should secondary school education be of the general education sort or should the present rigorous program be continued?

For me the situation was sharply outlined by a well-trained and well-traveled young man whom I met in Germany. This man is a civilian employee of our government, working in Germany with the occupation forces. He made the observation that it seemed to him, on the basis of his world-wide experiences with the army, that our educational program with all its faults produced a citizenry whose fund of information was better than that of any other country. That is, he implied that more of the young people of the United States were informed about government, science, mathematics, literature, and so on, than in any of the other countries in which he had worked. Now, this is encouraging. His statement made me, at least, reconsider our program of education. It is broad rather than narrow; it is universal rather than for the few; it does tend to neglect the gifted students and favor the average and below average. However, in our efforts to correct this fault, it would be unwise to sacrifice the good that comes out of our broad, general education program. The problem of the gifted student in mathematics and science must be solved within the context of our philosophy of the greatest amount of education for every child. What needs to be done in the next few years is to experiment with ways to develop to the fullest the potentialities of our gifted students as we have so successfully done for our average and slow-learning students.

Notes from the Washington office

by M. H. Ahrendt, Executive Secretary,
The National Council of Teachers of Mathematics, Washington, D. C.

Activities of the State Representatives

Almost from its beginning The National Council of Teachers of Mathematics has leaned upon a group of state representatives. The number of different state representatives who have served the Council during the thirty-four years of its existence is very great and our debt to them is incalculable.

The current school year is the fourth year during which the work of these representatives has been directed by the Executive Secretary through the Washington office. Because of the nature of its work, the Washington office has been a logical place in which to center this responsibility.

Each year the state representatives are asked to submit statistical and other reports of their activities. Reports for the 1953–54 school year were turned in by 43 of the 63 representatives. The work of the representatives falls into the following four categories. A summary of the work done in each category presents an interesting picture.

Selling memberships and subscriptions. Sales are usually best made through individual personal contacts. The representatives estimate that they made or directed a total of more than 6000 such contacts during the 1953–54 school year in an attempt to sell memberships in the Council.

Since the announcements of the establishment of the new journals, *The Arithmetic Teacher* and *The Mathematics Student Journal*, were made rather late in the school year, the representatives were asked to make up for the shortness of time by giving special attention to the promotion of these publications. Their response was good. They report that the new journals were announced at 148 different meetings to a total of more than 10,000 persons.

Announcing Council activities. The representatives are asked to advertise through announcements at meetings and in other ways the coming conventions of the Council, new publications, and other Council activities and projects. They report that announcements of this nature were made at 244 meetings to more than 14,000 persons.

Stimulate publicity in local and state publications. The representatives are asked to do all they can to have announcements of activities in and professional articles on the teaching of mathematics printed in newspapers and educational journals. They reported that announcements were made in more than 160 newspapers and journals. The representatives themselves motivated or prepared at least 67 of these. At least 90 professional articles were printed in local and state educational journals. The representatives motivated or prepared 18 of these.

Gather information. The representatives were not asked to gather any specific information during the past school year. However some of them did voluntarily turn in valuable information and suggestions. Activity of this nature is a valuable aid in helping us to "feel the pulse" of the Council.

The state representatives are busy people. Their work is a work of love. They get no pay and not much honor. Often they sacrifice other interests for the work of the Council. To them we owe much.

The representative, of course, cannot work without support. He must have the cooperation and assistance of his local and state colleagues. And in the occasional case in which the representative is inactive, he may need their stimulus and urging.

With the strengthening and expansion of the program of Affiliated Groups, it has become apparent that there needs to be a closer coordination of the efforts of the representatives with the work of the organized groups. In order to find new ways of building membership, a Committee on Membership has been appointed. Under the impetus of these new developments, we may find ways and means to revise and strengthen all of our promotion efforts.

There is one real danger implicit in the activities described above. Organized plans to solicit memberships and promote the interests of mathematics education may lead the rest of us to assume that the job is adequately being done by others. Actually the responsibility belongs to each of us. No professional organization can enjoy a healthy growth and expansion until each member designates himself as its representative.

State representatives of The National Council of Teachers of Mathematics, October 1, 1954

Mrs. Margaret M. Holland
Route 3, Box 728
Birmingham 8, Alabama

John B. Norman
Parket High School
Birmingham 4, Alabama

Lyle J. Dixon
Arkansas State College
State College, Arkansas

Paul Klipfel
485 Buena Vista Avenue
San Francisco 17, California

John S. Herman
Riverdale Joint Union High School
Riverdale, California

Dale Carpenter
1205 West Pico Boulevard
Los Angeles 15, California

Edwin Eagle
San Diego State College
San Diego 5, California

E. L. Vanderburgh
937 Veta Avenue
Pueblo, Colorado

Kenneth Fuller
Teachers College
New Britain, Connecticut

Russell Dineen
1804 No. Monroe Street
Wilmington, Delaware

Miss Veryl Schult
Sheraton Park Hotel
Washington 8, D. C.

Mrs. Ethel Harris Grubbs
751 Fairmont Street N. W.
Washington 1, D. C.

Kenneth P. Kidd
School of Education
University of Florida
Gainesville, Florida

Miss Bess Patton
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C. N. Fuqua
Champaign Senior High School
Champaign, Illinois

Philip Peak
Route 6
Bloomington, Indiana

Paul Shaw
Sloane Wallace Junior High School
Waterloo, Iowa

Gilbert Ulmer
University of Kansas
Lawrence, Kansas

Miss Bernice Wright
Western Ky. St. Teachers College
Bowling Green, Kentucky

Miss Iurnice Begnaud
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College of Arts and Sciences
Baton Rouge 3, Louisiana

Miss Katharine E. O'Brien
Deering High School
Portland, Maine

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Baltimore Polytechnic Institute
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Dearborn, Michigan

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Minneapolis 16, Minnesota

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Montana State College
Bozeman, Montana

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Exeter, New Hampshire

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Clovis, New Mexico

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Rochester, New York

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2624 Davidson Avenue
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Miss Clyde Hunter
Henderson High School
Henderson, North Carolina

Josephine Gustafson
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Devils Lake, North Dakota

Mrs. Frances Theaker
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Mansfield, Ohio

Miss Eunice Lewis
University High School
Norman, Oklahoma

Miss Eva Burkhalter
Klamath Union High School
Klamath Falls, Oregon

H. L. Krall
Pennsylvania State University
State College, Pennsylvania

Earle F. Myers
University of Pittsburgh
Pittsburgh, Pennsylvania

M. Albert Linton
Wm. Penn Charter School
Philadelphia, Pennsylvania

M. L. Herman
Moses Brown School
Providence, Rhode Island

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Ridgeland High School
Ridgeland, South Carolina

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Marjorie Linx
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John G. Bowker
Middlebury College
Middlebury, Vermont

Miss Wilhelmina Wright
John Marshall High School
Richmond 19, Virginia

R. W. James
Armstrong High School
North Thirty-First Street
Richmond 23, Virginia

Harold J. Hunt
515 N. 50th Street
Seattle 3, Washington

Fred Kramlich
Lewis & Clark High School
Spokane, Washington

Mrs. Margaret E. Holmes
110-17th Street
Dunbar, West Virginia

Mrs. Maggie W. Powell
110 East 12th Street
Parkersburg, West Virginia

Miss Elli Otteson
705 Whipple
Eau Claire, Wisconsin

Miss Margaret Joseph
1504 N. Prospect Avenue
Milwaukee 2, Wisconsin

Palmer O. Steen
University of Wyoming
Laramie, Wyoming

James W. Kerr
Leaside High School
Hanna Road
Leaside, Ontario, Canada

Annual business meeting

As required by the by-laws, notice is hereby given to the members of The National Council of Teachers of Mathematics that the annual business meeting will be held at 4:30 P.M. Friday, April 16, 1955, at the Hotel Statler, Boston, Massachusetts.

Your professional dates

The information below gives the date, name, and place of meeting, with the name and address of the person to whom you may write for further information. For information about other meetings, see the previous issues of THE MATHEMATICS TEACHER. Announcements for this column should be sent at least ten weeks early to the Executive Secretary, National Council of Teachers of Mathematics, 1201 Sixteenth Street N.W., Washington 6, D. C.

NCTM convention dates

April 13–16, 1955
ANNUAL MEETING
Statler Hotel, Boston, Massachusetts
Jackson Adkins, local chairman, Phillips Exeter Academy, Exeter, New Hampshire

July 4, 1955
JOINT MEETING WITH NEA
Chicago, Illinois
E. H. C. Hildebrandt, local chairman, Northwestern University, Evanston, Illinois

August 21–24, 1955
SUMMER MEETING
Indiana University, Bloomington, Indiana
Philip Peak, local chairman, Indiana University, Bloomington, Indiana

December 27–30, 1955
CHRISTMAS MEETING
Sheraton-Park Hotel, Washington, D. C.
Veryl Schult, local chairman, Wilson Teachers College, Washington 9, D. C.

Other professional dates

March 18, 1955
Women's Mathematics Club of Chicago and Vicinity in Joint Meeting with Men's Mathematics Club
Western Society of Engineers, 84 E. Randolph Street, Chicago, Illinois
Mary E. Hradek, 5301 S. Rockwell Street, Chicago 32, Illinois

March 19, 1955
Second Annual Workshop of the Mathematics Teachers Association of Western Pennsylvania
University of Pittsburgh, Pittsburgh, Pennsylvania
Catherine A. V. Lyons, 12 S. Fremont Avenue, Pittsburgh 2, Pennsylvania

March 26, 1955
Eighth Annual Conference on the Teaching of Elementary and Secondary Mathematics
Illinois State Normal University, Normal, Illinois
T. E. Rine, ISNU Mathematics Conference, Illinois State Normal University, Normal, Illinois

April 13–14, 1955
Mathematics Conference of the Ontario Association of Teachers of Mathematics and Physics
University of Toronto, Toronto, Ontario
Ern Totton, Forest Hill Collegiate Institute, Toronto, Ontario

April 29–30, 1955
Fifth Annual Spring Conference of the Ohio Council of Teachers of Mathematics
Kent State University, Kent, Ohio
Mildred Keiffer, Cincinnati Public Schools, Cincinnati, Ohio

April 29–May 1, 1955
Sixth Annual Conference of the Michigan Council of Teachers of Mathematics
St. Mary's Lake Camp, Battle Creek, Michigan
Irene Smith, Ann Arbor High School, Ann Arbor, Michigan

April 30, 1955
Spring Meeting of Colorado Council of Teachers of Mathematics
Pueblo Junior College, Pueblo, Colorado
Mrs. Amanda Lindsey, 753 S. Sherman Street, Denver 9, Colorado

May 6–7, 1955
Annual Meeting of Association of Mathematics Teachers of New York State
Hotel Syracuse, Syracuse, New York
Pauline A. Morris, Geneva High School, Geneva, New York

May 21, 1955
Annual Spring Meeting of the Indiana Council of Teachers of Mathematics
Butler University, Indianapolis, Indiana
R. L. Green, 3401 N. Meridian, Indianapolis, Indiana

June 13–July 22, 1955
National Science Foundation Collegiate Conference
Oklahoma A and M College, Stillwater, Oklahoma
L. Wayne Johnson, Head of Department of Mathematics, Oklahoma A and M College, Stillwater, Oklahoma

July 6–15, 1955
Third New Jersey Mathematics Institute
Rutgers University, New Brunswick, New Jersey
Director of the Summer Session, Rutgers University, New Brunswick, New Jersey

August 18–25, 1955
Seventh Annual Mathematics Institute of the Association of Teachers of Mathematics in New England
Middlebury College, Middlebury, Vermont
Harriet Howard, Ethel Walker School, Simsbury, Connecticut

Report of the Nominating Committee

The Committee on Nominations and Elections presents the persons listed below as candidates for office of Vice-President, College Level; Vice-President, Junior High School Level; and additional members of the Board of Directors. There are two candidates for each position.

The term for a vice-president is two years and he is not eligible for re-election. The term for a director is three years. Directors are eligible for re-election.

The Committee wishes to express its appreciation and thanks to those members of the NCTM, and the Affiliated Groups who made nominations for the various offices. Many able individuals could not be given recognition because someone from their state was a member of the Board. The Committee found it difficult to determine which of all the capable individuals whose names were submitted should be nominated. The Committee regrets that recognition by nomination could not be given all.

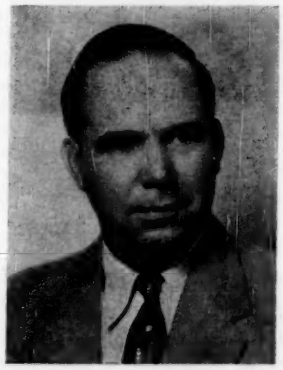
Election ballots will be mailed February 25, 1955, to all persons who are members on that date and, also, to those whose membership orders are postmarked not later than February 25, 1955.

Please mark ballots, and return promptly. Ballots must be postmarked not later than March 25, 1955, to be valid.

AGNES HERBERT, *Chairman*
JACKSON ADKINS
CLIFFORD BELL
HARRY CHARLESWORTH
MAURICE HARTUNG
HOUSTON KARNES
C. V. NEWSOM
MARY A. POTTER
MARY C. ROGERS

NOMINEES FOR VICE-PRESIDENT—COLLEGE LEVEL

FRANCIS G. LANKFORD, JR.



BRUCE E. MESERVE

Francis G. Lankford, Jr.

Professor of Education, University of Virginia.

B.S., Randolph-Macon College 1928; M.S., 1932; Ph.D., 1938, University of Virginia; General Education Fellow for Post-doctoral study at University of Michigan 1939–40; also graduate work at the University of Chicago.

Taught mathematics in high schools in Virginia. Critic teacher in mathematics at University of Virginia and currently teacher of graduate and undergraduate classes in mathematics education. Director of summer institutes for mathematics teachers at Virginia and for several years has directed a mid-winter conference for Virginia mathematics teachers.

Co-author of several mathematics books,

and departmental editor of "Tips for Beginners" in THE MATHEMATICS TEACHER.

Formerly, Virginia State Representative of the National Council of Teachers of Mathematics and chairman of its yearbook planning committee.

Bruce E. Meserve

Associate Professor of Mathematics, New Jersey State Teachers College at Montclair.

A.B., Bates College; A.M. and Ph.D., Duke University.

Teacher of Mathematics, Moses Brown School 1938–41; Graduate work at Duke University 1941–42 and 1945–46; Civilian Public Service 1942–43; U. S. Army 1943–45; Instructor of Mathematics, University of Illinois 1946–48, Assistant Professor 1948–54; Associate Professor of Mathematics, New Jersey State Teachers College at Montclair 1954–.

Member NCTM; Member of Committee on Research, 1953–54, Chairman, 1954–55; Secretary of Committee on Coordination of Mathematics with Business and Industry, 1954–55; member of editorial committee for forthcoming NCTM yearbook on the interpretation of modern mathematics for high school teachers.

Member of Illinois Council of Teachers of Mathematics, vice-president for colleges, 1952–54; Chairman of Committee on the Strengthening of the Teaching of Mathematics, 1953–54; member of Association of Teachers of Mathematics in New England; Association of Mathematics Teachers of New Jersey; Mathematical Association of America; American Mathematical Society; member of Committee on Contests and Awards of Metropolitan New York Section of the Mathematical Association of America; Member of Phi Beta Kappa, Sigma Xi, and Kappa Mu Epsilon.

Author of *Fundamental Concepts of Algebra* and *Fundamental Concepts of Geometry*. Co-author of the University of Illinois Bulletin entitled "Mathematical Needs of Prospective Students at the College of Engineering of the University of Illinois." Author of eleven articles (two jointly with other authors) in THE MATHEMATICS TEACHER (6) and five other mathematical journals.

NOMINEES FOR VICE-PRESIDENT—JUNIOR HIGH SCHOOL LEVEL

MILTON W. BECKMANN

DONALD W. LENTZ

Milton W. Beckmann

Supervisor of Mathematics and Assistant Professor, Teachers College High School, University of Nebraska.

B.A. and M.A., University of Nebraska; additional study, University of Chicago; Ph.D., University of Nebraska.

Activities in NCTM include: President, Nebraska Section, NCTM, 1940, 1950–51, 1951–52; General Chairman for Thirteenth Christmas Meeting of NCTM in Lincoln, 1952; appearance on its convention programs; member of Committee on Evaluation of Films and Filmstrips; Chairman of the Subcommittee on Educational Journals of the National Council's Publicity Committee.

Teacher of junior high mathematics, Kearney, Nebraska, for five years; high school mathematics teacher; administrator of schools for seven years; Supervisor of Mathematics, Teachers College High School, University of Nebraska, 1949–. Life member, Nebraska State Education Association (President, District No. 4, 1948–49); member, Nebraska Schoolmasters Club (Secy.-Treas.); "N" Club; NEA; Pi Mu Epsilon; Phi Delta Kappa; AAUP; Capital City Education Association, Lincoln (President, 1952–53); active participant in local church, community, and fraternal activities.

Publications include: "How Mathematically Literate Is the Typical Ninth Grader After Having Completed Either General Mathematics

or Algebra?" *School Science and Mathematics*, June, 1952; "Teaching Safety Through Mathematics," *Safety Education*, 1942; "Safety Education Needs Met Mathematically," *School Science and Mathematics*, December, 1941; author of *Shop Mathematics*, correspondence course, Extension Division, University of Nebraska; "Is General Mathematics or Algebra Providing Greater Opportunity to Attain the Recommended Mathematical Competencies?" *Journal of Educational Research*, March, 1954; Educational author of the filmstrip and guide, "Literacy in Mathematics"; *Developing Mathematical Literacy in Nebraska's Youth*, 1954; Arithmetic book for the United States Armed Forces Institute, 1954.

Donald W. Lentz

Principal, Schaaf Junior High School, Parma, Ohio.

B.S., Case Institute of Technology; LL.B., Western Reserve University; M.A., Western Reserve University; University of Colorado, Columbia University.

Teacher of Mathematics, Rawlings Junior High School, Cleveland, Ohio, 1938–45 (Chairman of Mathematics Department, 1941–45); Teacher of mathematics, West Technical High School and John Hay High School, 1939–1943 (summer and evening extension program); Principal, Ridge Junior High School, Parma, Ohio, 1945–53; Principal, Schaaf Junior High School, Parma, Ohio, 1953–54.

Activities in NCTM include: Member of Committee on Cooperation between Mathematics and Industry, 1953–54; Chairman, Committee on Improving Cooperation between NCTM and Future Scientists of America Foundation.

Membership: National Council of Teachers of Mathematics (on convention program, Cincinnati, St. Louis, 1954), Central Association of Science and Mathematics Teachers (Past President, Board of Directors, Chairman of Nominating Committee), Past member of the Cooperative Committee on the Teaching of Science and Mathematics of the American Association for the Advancement of Science, Past President, Greater Cleveland Mathematics Club, National Education Association, Ohio Education Association, North Eastern Ohio Teachers Association, Cuyahoga County School Administrators, National Association of Secondary School Principals, Honorary Life Member of Family Service Association, Theta Chi.

Co-author of *Everyday Arithmetic*, Books I, II. Author of several articles in *School Science and Mathematics* and "The Mathematics Program from the Viewpoint of the High School Principal," *National Association of Secondary School Principals Journal.*

NOMINEES FOR THE BOARD OF DIRECTORS

Jackson B. Adkins

The Phillips Exeter Academy, Exeter, New Hampshire.

Ph.B., University of Chicago; Ed.M., Harvard University. Teacher of Mathematics, Central Junior High School, Lima, Ohio,, 1923–25; Bloom Township High School, Chicago Heights, Illinois, 1926–27; Proviso Township High School, Maywood, Illinois, 1927–29; Culver Military Academy, Culver, Indiana, 1929–32; Moses Brown School, Providence, Rhode Island, 1933–39; Phillips Exeter Academy, Exeter, New Hampshire, 1939–. Teacher, U. S. Navy Midshipman School, New York, 1942–44; Head of Mathematics Department, U. S. Navy Premidshipman School, Asbury Park and Princeton, New Jersey, 1944–45.

Member of NCTM (Member of Committee on the Official Journal, Committee on Affiliated Groups). Member Am. Math. Assn., Member of Association of Teachers of Mathematics in New England (Board of Directors, 1947–50, '51–'53, Speakers Bureau, 1951–52, Vice-president, 1952–54, President, 1954–); Program Chairman, New England Institute for Teachers of Mathematics, 1951; General Chairman of the Joint Meeting of the Institute and the NCTM, 1952; Chairman of Local Arrangements for the spring meeting of the NCTM in Boston, 1955. Founder and President of Southeastern New Hampshire Association of Teachers of Mathematics, 1946–. New Hampshire State Committee on the Revision of the Senior High School Curriculum in Mathematics. Instructor, State Teachers College, Plymouth, New Hampshire, July, 1954. Co-author, *A First Course in Algebra*. Phi Delta Kappa; Tau Kappa Epsilon.

Ida May Bernhard

Consultant in Secondary Education, Texas Education Agency (State Department of Education), Austin, Texas.

A.B., M.A., University of Texas; summer sessions at University of Vermont and Teachers College, Columbia University.

Teacher in Texas Public Schools, 1927–45; Supervisor of Mathematics in Laboratory School, Southwest Texas State Teachers College and San Marcos High School, San Marcos, Texas, 1945–52. Attended Duke University Mathematics Institute, 1946–52. Study Group Leader, Duke University Mathematics Institute, 1950–52; Louisiana State University Mathematics Institute, 1950–52, 1954; U.C.L.A. Annual California Conference for Teachers of Mathematics, 1951–1954; University of Houston

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HENRY SWAIN

Mathematics Institute, 1952–54; University of Texas Mathematics Conference, 1953. Consultant, Texas State College for Women Mathematics Workshop, 1953–54.

Member of NCTM; NEA; Mathematics Association of America; Texas State Teachers Association (Vice-President, Alamo District II, 1951–52; President, Alamo District II, 1952–53); State Honorary Member of Delta Kappa Gamma; Texas State Textbook Committee, 1951; Texas Association of Supervision and Curriculum Development; Texas Association of Elementary Principals; Texas Council of Teachers of Mathematics. Activities in the NCTM include Member of Board of Directors, 1952–55; Southwestern Area Representative for Affiliated Groups Committee, 1952–55; appearance on convention programs; participation on committees.

Author of "Materials Available for Counseling in Mathematics," THE MATHEMATICS TEACHER, April 1954. Served as leader of committee which prepared Texas Education Agency Bulletin 548, *Suggestions for Teachers of Mathematics*, 1953.

John A. Brown

Teacher of Mathematics, Wisconsin High School, 1949–55; Instructor, Department of Education, and Assistant, Department of Mathematics, University of Wisconsin, Madison, Wisconsin, 1954–55.

B.S., La Crosse State College; M.S., The University of Wisconsin; attended the Naval Training School, Harvard University, 1943.

Teacher of Mathematics, High School, Mattoon, Wisconsin; Oconto, Wisconsin; Merrill, Wisconsin.

NCTM Activities: Co-editor with H. T. Karnes of the department in THE MATHEMATICS TEACHER, "What Is Going on in Your School?" 1952–55; chairman, Committee on Distribution of Free Materials, Delegate to Delegate Assembly, 1953; co-author of chapter with J. R. Mayor in the Twenty-second Yearbook; Member of Committee on Revision of the Guidance Pamphlet, 1953–54; Chairman, Committee on the Mathematics Number of the *N.A.S.S.P. Bulletin*, May 1954. Wisconsin Mathematics Council Activities: Member of Research Committee;

Member of Committee on Geometry; Vice-president, Junior High School level, 1952–53.

Member of Wisconsin Statewide Committee on Mathematics Curriculum, 1954–. Assistant Director, Summer Mathematics Conferences, University of Wisconsin, 1953 and 1954; Consultant, Summer Mathematics Conference, University of Nebraska, 1954.

Author of correspondence course in Plane Geometry, University of Wisconsin Extension Division.

Member of the U. S. Naval Reserve.

Mildred Keiffer

A.B. and B.E., University of Cincinnati; M.A., Teachers College, Columbia University. Junior High School Teacher, 1934–35; Senior High School Teacher, 1936–47.

Attendance at various workshops; Duke, New York University, Rutgers, and the University of Wisconsin. Member of innumerable committees on courses of study, textbooks, general education, etc. Served as general chairman of local arrangements for the 32nd annual meeting of NCTM held in Cincinnati, 1954. President of the Ohio Council of Teachers of Mathematics; Past President of the Mathematics Group of Greater Cincinnati; Program Chairman of the Ohio Council, 1952.

Member: NCTM, CASMT, AAAS, ASCD, ACEI, Phi Beta Kappa, Kappa Delta Pi, Delta Kappa Gamma, and life member of NEA.

Philip Peak

Assistant to the Dean, School of Education, and Head of the Mathematics Department, University School, Indiana University, Bloomington, Indiana.

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NCTM activities include: Indiana State Representative; Member of Twenty-second Yearbook Committee; Chairman of Research in Algebra Committee; and Secretary for the Madison, Wisconsin, Mathematics Symposium, 1952.

High school Mathematics Teacher, Mechanicsville, Ia.; Head of Mathematics Department, high school, Pierre, S. D., Assistant Professor of Mathematics and Critic Teacher, Chadron State Teachers College, Chadron, Nebraska; Head of Mathematics Department, University School, Indiana University, 1942–, and Assistant to the Dean, School of Education, Indiana University, 1950–. Member, CASMT (President, 1951–52); AAUP; NEA; Indiana State Teachers Association (Past Chairman, Mathematics Section); Phi Delta Kappa (Past Treasurer of Alpha Chapter). Associate Editor, THE MATHEMATICS TEACHER, and author of the column, "Have You Read?" in that publication each month. Author of articles on the teaching of mathematics in THE MATHEMATICS TEACHER, *School Science and Mathematics*, *School and Society* and *Phi Delta Kappan*.

Henry Swain

New Trier Township High School, Winnetka, Illinois.

A.B., Swarthmore College; M.A., Lehigh University; additional graduate study at Princeton University, University of Chicago, and Purdue University (G.E. Fellow 1953).

Instructor in Mathematics, Lehigh University, 1930–1934; Princeton University, 1934–1936; The Hill School, Pottstown, Pennsylvania, 1936–1939; Concord State Teachers College, Athens, West Virginia, 1939–1940; Evanston Township High School, Evanston, Illinois, 1940–1941; New Trier Township High School, 1941–; part-time instructor, Northwestern University, 1947–.

Member, Illinois Council of Teachers of Mathematics (Vice-President, 1949–50; Membership Chairman, 1949–52; President, 1952–53). Men's Mathematics Club of Chicago (Program Chairman, 1950–51; Recording Secretary, 1952–53). Illinois Committee on the Strengthening of the Teaching of Mathematics, 1952–. Phi Beta Kappa.

Activities in National Council of Teachers of Mathematics include: Chairman of Equipment Committee for annual meeting in Chicago, 1950; State Representative for Illinois, 1950–52; Membership Committee, 1952–53; Delegate to Delegate Assembly, 1951, 1952, 1953; Committee on Publications of Current Interest, 1951–; wrote pamphlet on "How to Study Mathematics"; reviews for Book Section of THE MATHEMATICS TEACHER; editor of *Handbook for Affiliated Groups*.

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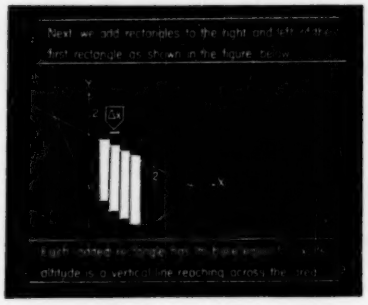

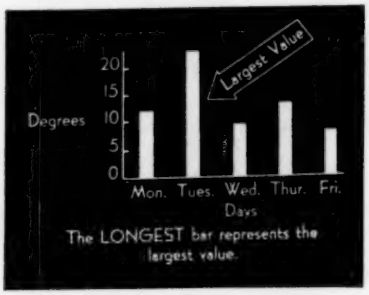

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